Heterogeneous formation control in presence of noisy measurements

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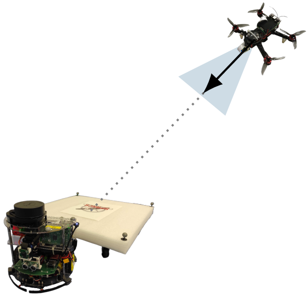
ECC 2022 WS – Rigidity Theory applied to Dynamic Systems: from Parallel Robots to Multi-Agent Formations

Motivation

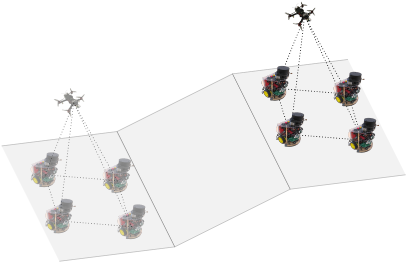
Estimation and control of heterogeneous multi-agent systems via rigidity theory in realistic scenarios

UAV on AGV Landing



CCTA22

UAV-AGV Formation Control



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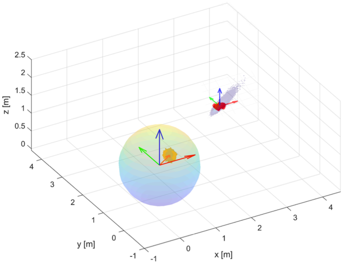

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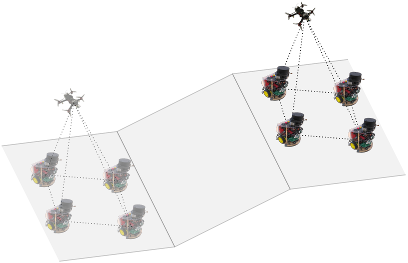

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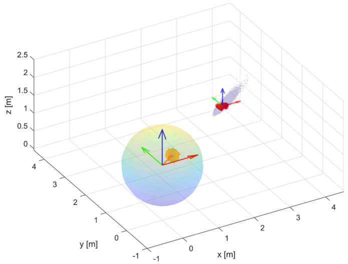

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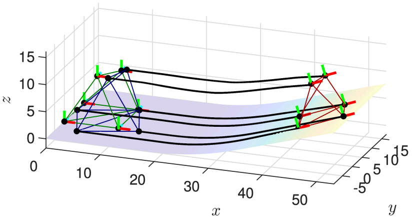

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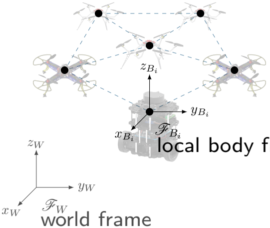
Multi-agent system modeling & rigidity theory

Multi-agent System Modeling - 1

Model of agent system

n	fully actuated agents
$x_i = (\mathbf{p}_i, \mathbf{R}_i)$ $x_i \in \mathbb{SE}(3)$	i-th agent pose w.r.t. $\mathscr{F}_W$
$u_i = (\mathbf{v}_i, \boldsymbol{\omega}_i)$ $u_i \in \mathbb{R}^3 \times \mathbb{R}^3$	i-th agent actuation w.r.t. $\mathscr{F}_B$



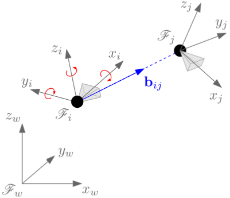
local body frame

world frame

unavailable to group

- agents sensing capabilities

$$\mathbf{b}_{ij} = \mathbf{R}_i^\top \bar{\mathbf{p}}_{ij} \in \mathbb{S}^2$$

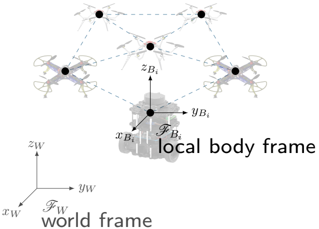


where

$$\bar{\mathbf{p}}_{ij} = \frac{\mathbf{p}_j - \mathbf{p}_i}{\|\mathbf{p}_j - \mathbf{p}_i\|} = \frac{1}{d_{ij}}(\mathbf{p}_j - \mathbf{p}_i) \in \mathbb{R}^3$$

Multi-agent System Modeling - 2

Model of multi-agent system



local body frame

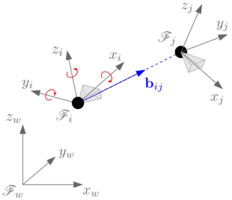
world frame

unavailable to group

n	fully actuated agents
$\mathbf{x} \in \mathbb{SE}(3)^n$	system state w.r.t. $\mathscr{F}_W$
$\mathbf{u} \in \mathbb{R}^{3n} \times \mathbb{R}^{3n}$	system actuation w.r.t. $\mathscr{F}_B$

$$\dot{\mathbf{x}} = \mathbf{D}(\mathbf{x})\mathbf{u}$$

- agents sensing capabilities



Given a n-agent formation modeled as a framework $(\mathcal{G}, \mathbf{x})$ in $\mathbb{SE}(3)^n$, the bearing rigidity function is the map $\mathbf{b}_{\mathcal{G}} : \mathbb{SE}(3)^n \rightarrow \mathbb{S}^{2m}$ such that

$$\mathbf{b}_{\mathcal{G}}(\mathbf{x}) = \left[\mathbf{b}_1^\top \dots \mathbf{b}_m^\top\right]^\top$$

$\mathbf{b}_k,\ k \in \{1 \dots m\}$, denotes the measurement on the k-th directed edge in $\mathcal{G}$, chosen any labeling.

Rigidity Matrix

Dynamics of bearing function:

$$\dot{\mathbf{b}}_{\mathcal{G}}(\mathbf{x}) \;=\; \underbrace{\mathbf{B}_{\mathcal{G}}(\mathbf{x})}_{\text{bearing rigidity matrix}}\mathbf{u}$$

- $\dot{\mathbf{b}}_{\mathcal{G}}(\mathbf{x}) = \mathbf{B}_{\mathcal{G}}(\mathbf{x})\mathbf{u}$ $\Rightarrow$

infinitesimal variation	$\mathbf{u} \in \ker \mathbf{B}_{\mathcal{G}}(\mathbf{x})$
trivial variation	$\mathbf{u} \in \ker \mathbf{B}_{\mathcal{K}}(\mathbf{x})$

A given formation modeled as a framework $(\mathcal{G}, \mathbf{x})$ in $\mathbb{R}^3 \times \mathbb{S}^3$ is IBR if and only if

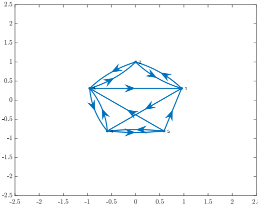
$$\ker(\mathbf{B}_{\mathcal{G}}(\mathbf{x})) = \ker(\mathbf{B}_{\mathcal{K}}(\mathbf{x})),$$

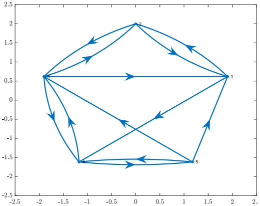
$$\text{rank}(\mathbf{B}_{\mathcal{G}}(\mathbf{x})) = 6n - 7,$$
$$\lambda_8(\mathbf{B}_{\mathcal{G}}^T(\mathbf{x})\mathbf{B}_{\mathcal{G}}(\mathbf{x})) > 0$$

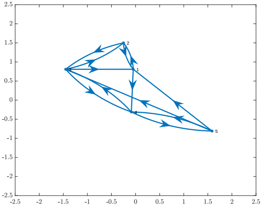
where $\mathcal{K}$ is the complete graph associated to the graph $\mathcal{G}$.

Rigidity of a Configuration

Given a multi-agent system:
can we "measure the rigidity property"?







Rigidity of a Configuration

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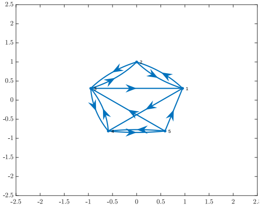
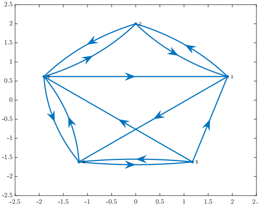
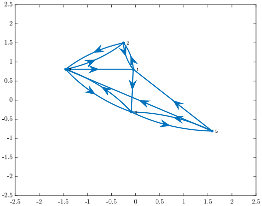
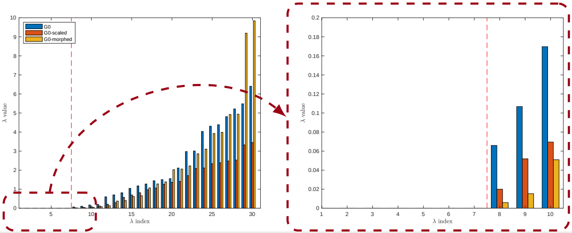




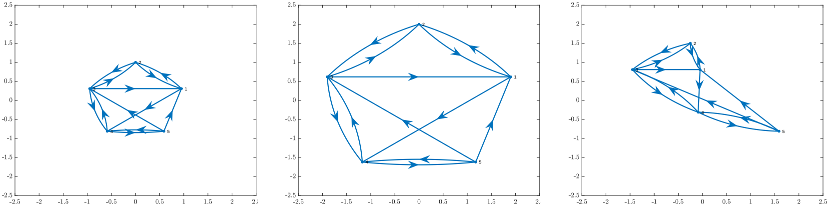


- eigenvalue analysis of the symmetric rigidity matrix...



Rigidity of a Configuration

**Given a multi-agent system:
can we "measure the rigidity property"?**



Take-home message: the "rigidity" of a multi-agent system appears to imply more than a binary property.

$(6n - 7)$ Shades of Rigidity

$(6n-7)$ Shades of Rigidity

$$\mathbf{B}_{\mathcal{G}}(\mathbf{x}) = \left[\begin{array}{ccccccc} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathbf{0}_{3\times 3(i-1)} & -\mathbf{Q}(d_{ij},\mathbf{b}_{ij}) & \mathbf{0}_{3\times 3(j-i-1)} & \mathbf{Q}(d_{ij},\mathbf{b}_{ij})\mathbf{R}_i^\top\mathbf{R}_j & \mathbf{0}_{3\times 3(j-i-1)} & [\mathbf{b}_{ij}]_\times & \mathbf{0}_{3\times 3(n-i)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & \downarrow & & \downarrow & & \downarrow & \\ & \mathbf{v}_i & & \mathbf{v}_j & & \boldsymbol{\omega}_i & \end{array}\right]$$

$\mathbf{B}_{\mathcal{G}}(\mathbf{x}) \in \mathbb{R}^{3m\times 6n}$

For each k-th edge, there appear three terms:

- term related to the position of i w.r.t. j
- term related to the position of j w.r.t. i
- term related to the bearing orientation of j w.r.t. i

$$-\mathbf{Q}(d_{ij},\mathbf{b}_{ij}) = -\frac{1}{d_{ij}}\mathbf{P}(\mathbf{b}_{ij}) \qquad \mathbf{Q}(d_{ij},\mathbf{b}_{ij})\mathbf{R}_i^\top\mathbf{R}_j = -\frac{1}{d_{ij}}\mathbf{P}(\mathbf{b}_{ij})\mathbf{R}_i^\top\mathbf{R}_j$$

$(6n-7)$ Shades of Rigidity

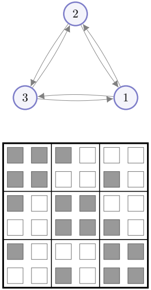
$$\mathbf{B}_{\mathcal{G}}(\mathbf{x}) = \begin{array}{cc} \begin{bmatrix} -\mathbf{Q}(d_{12},\mathbf{b}_{12}) & [\mathbf{b}_{12}]_\times & \mathbf{Q}(d_{12},\mathbf{b}_{12})\mathbf{R}_1^\top\mathbf{R}_2 & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ -\mathbf{Q}(d_{13},\mathbf{b}_{13}) & [\mathbf{b}_{13}]_\times & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{Q}(d_{13},\mathbf{b}_{13})\mathbf{R}_1^\top\mathbf{R}_3 & \mathbf{0}_{3\times3} \\ \mathbf{Q}(d_{21},\mathbf{b}_{21})\mathbf{R}_2^\top\mathbf{R}_1 & \mathbf{0}_{3\times3} & -\mathbf{Q}(d_{21},\mathbf{b}_{21}) & [\mathbf{b}_{21}]_\times & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & -\mathbf{Q}(d_{23},\mathbf{b}_{23}) & [\mathbf{b}_{23}]_\times & \mathbf{Q}(d_{23},\mathbf{b}_{23})\mathbf{R}_2^\top\mathbf{R}_3 & \mathbf{0}_{3\times3} \\ \mathbf{Q}(d_{31},\mathbf{b}_{31})\mathbf{R}_3^\top\mathbf{R}_1 & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & -\mathbf{Q}(d_{31},\mathbf{b}_{31}) & [\mathbf{b}_{31}]_\times \\ \mathbf{0}_{3\times3} & \mathbf{0}_{3\times3} & \mathbf{Q}(d_{32},\mathbf{b}_{32})\mathbf{R}_3^\top\mathbf{R}_2 & \mathbf{0}_{3\times3} & -\mathbf{Q}(d_{32},\mathbf{b}_{32}) & [\mathbf{b}_{32}]_\times \end{bmatrix} & \begin{array}{c} \rightarrow e_{12} \\ \rightarrow e_{13} \\ \rightarrow e_{21} \\ \rightarrow e_{23} \\ \rightarrow e_{31} \\ \rightarrow e_{32} \end{array} \\ \begin{array}{cccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ \mathbf{v}_1 & \boldsymbol{\omega}_1 & \mathbf{v}_2 & \boldsymbol{\omega}_2 & \mathbf{v}_3 & \boldsymbol{\omega}_3 \end{array} & \end{array}$$

By reshaping the matrix (e.g. $n=3$, $m=6$) there appear three blocks:

- term related to node n_1: sensing and actuation
- term related to node n_2: sensing and actuation
- term related to node n_3: sensing and actuation

$(6n-7)$ Shades of Rigidity

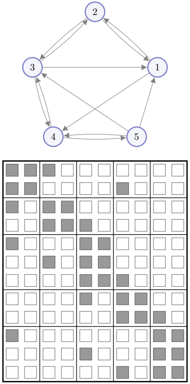
$n = 3 \quad m = 6$



In "terms of mini-blocks":

$$\mathbf{B}_{\mathcal{G}}(\mathbf{x}) \in \mathbb{R}^{6\times 6}$$
$$\text{nz} = 6 \times 3 = 18$$

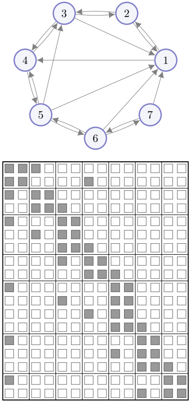
$n = 5 \quad m = 12$



In "terms of mini-blocks":

$$\mathbf{B}_{\mathcal{G}}(\mathbf{x}) \in \mathbb{R}^{12\times 10}$$
$$\text{nz} = 12 \times 3 = 36$$

$n = 7 \quad m = 18$



In "terms of mini-blocks":

$$\mathbf{B}_{\mathcal{G}}(\mathbf{x}) \in \mathbb{R}^{18\times 14}$$
$$\text{nz} = 18 \times 3 = 54$$

$(6n-7)$ Shades of Rigidity

$$\mathbf{B}_{\mathcal{G}}(\mathbf{x})^\top \mathbf{B}_{\mathcal{G}}(\mathbf{x}) = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & 0_{3\times 3} & (2,5) & 0_{3\times 3} \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & 0_{3\times 3} & (4,3) & (4,4) & (4,5) & 0_{3\times 3} \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & 0_{3\times 3} & (6,3) & 0_{3\times 3} & (6,5) & (6,6) \end{bmatrix}$$

- point of view of *nodes*...

$\mathbf{B}_{\mathcal{G}}^\top(\mathbf{x})\mathbf{B}_{\mathcal{G}}(\mathbf{x}) \in \mathbb{R}^{6n\times 6n}$

symmetric rigidity matrix

$$(1,1) = \mathbf{Q}(d_{12},\mathbf{b}_{12})^2 + \mathbf{Q}(d_{13},\mathbf{b}_{13})^2 + \left(\mathbf{Q}(d_{21},\mathbf{b}_{21})\mathbf{R}_2^\top \mathbf{R}_1\right)^2 + \left(\mathbf{Q}(d_{31},\mathbf{b}_{31})\mathbf{R}_3^\top \mathbf{R}_1\right)^2$$

$$(1,2) = -\mathbf{Q}(d_{12},\mathbf{b}_{12})[\mathbf{b}_{12}]_\times - \mathbf{Q}(d_{13},\mathbf{b}_{13})[\mathbf{b}_{13}]_\times$$

$$(2,2) = [\mathbf{b}_{12}]_\times^2 + [\mathbf{b}_{13}]_\times^2$$

$$(1,3) = -\mathbf{Q}(d_{12},\mathbf{b}_{12})\mathbf{Q}(d_{12},\mathbf{b}_{12})\mathbf{R}_1^\top \mathbf{R}_2 - \mathbf{Q}(d_{21},\mathbf{b}_{21})\mathbf{R}_2^\top \mathbf{R}_1 \mathbf{Q}(d_{21},\mathbf{b}_{21})$$

$$(1,4) = \mathbf{Q}(d_{21},\mathbf{b}_{21})\mathbf{R}_2^\top \mathbf{R}_1[\mathbf{b}_{21}]_\times$$

$$(2,3) = [\mathbf{b}_{12}]_\times \mathbf{Q}(d_{12},\mathbf{b}_{12})\mathbf{R}_1^\top \mathbf{R}_2$$

$(6n-7)$ Shades of Rigidity

$$\mathbf{B}_{\mathcal{G}}(\mathbf{x})^\top \mathbf{B}_{\mathcal{G}}(\mathbf{x}) = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & \mathbf{0}_{3\times 3} & (2,5) & \mathbf{0}_{3\times 3} \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & \mathbf{0}_{3\times 3} & (4,3) & (4,4) & (4,5) & \mathbf{0}_{3\times 3} \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & \mathbf{0}_{3\times 3} & (6,3) & \mathbf{0}_{3\times 3} & (6,5) & (6,6) \end{bmatrix}$$

- point of view of *nodes*...

$\mathbf{B}_{\mathcal{G}}^\top(\mathbf{x})\mathbf{B}_{\mathcal{G}}(\mathbf{x}) \in \mathbb{R}^{6n\times 6n}$

symmetric rigidity matrix

- $\dot{\mathbf{b}}_{\mathcal{G}}(\mathbf{x}) = \mathbf{B}_{\mathcal{G}}(\mathbf{x})\mathbf{u} \quad \Rightarrow$

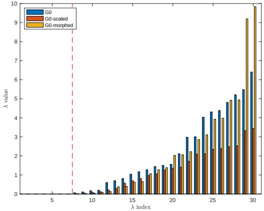
$$\begin{aligned} \|\dot{\mathbf{b}}_{\mathcal{G}}(\mathbf{x})\|^2 &= \mathbf{u}^\top \mathbf{B}_{\mathcal{G}}^\top(\mathbf{x})\mathbf{B}_{\mathcal{G}}(\mathbf{x})\mathbf{u} \\ &= \|\mathbf{u}\|^2_{\mathbf{B}_{\mathcal{G}}^\top(\mathbf{x})\mathbf{B}_{\mathcal{G}}(\mathbf{x})} \end{aligned}$$

Take-home message: the bearing rigidity matrix acts as a gain between the "motions" (control) and the "observation" variations (measurements).

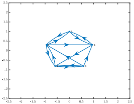
$(6n-7)$ Shades of Rigidity

$$\begin{aligned}\|\mathbf{B}_{\mathcal{G}}(\mathbf{x})\|_F^2 &= \mathrm{trace}(\mathbf{B}_{\mathcal{G}}^\top(\mathbf{x})\mathbf{B}_{\mathcal{G}}(\mathbf{x})) = \mathrm{trace}(\mathbf{B}_{\mathcal{G}}(\mathbf{x})\mathbf{B}_{\mathcal{G}}^\top(\mathbf{x})) \\ &= \sum \sigma^2_{\mathbf{B}_{\mathcal{G}(\mathbf{x})},i} = \sum \lambda_{\mathbf{B}_{\mathcal{G}}^\top(\mathbf{x})\mathbf{B}_{\mathcal{G}}(\mathbf{x}),i}\end{aligned}$$

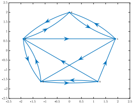
$$\dot{\mathbf{b}}_{\mathcal{G}}(\mathbf{x}) = \mathbf{B}_{\mathcal{G}}(\mathbf{x})\mathbf{u} \qquad \Rightarrow \qquad \mathbf{u} = \mathbf{B}_{\mathcal{G}}^\dagger(\mathbf{x})\dot{\mathbf{b}}_{\mathcal{G}}(\mathbf{x})$$



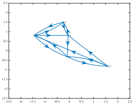
$\sigma_8 = 0.066 \quad \|\mathbf{B}_{\mathcal{G}}(\mathbf{x})\|_F = 7.305$



$\sigma_8 = 0.020 \quad \|\mathbf{B}_{\mathcal{G}}(\mathbf{x})\|_F = 5.598$



$\sigma_8 = 0.006 \quad \|\mathbf{B}_{\mathcal{G}}(\mathbf{x})\|_F = 7.425$



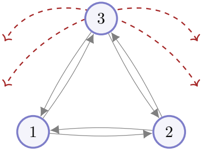
$(6n-7)$ Shades of Rigidity: the 3-agent case

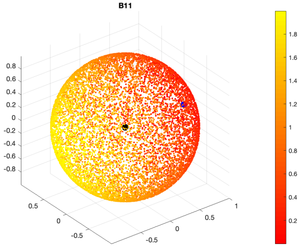
- eigenvalue analysis of the symmetric rigidity matrix...

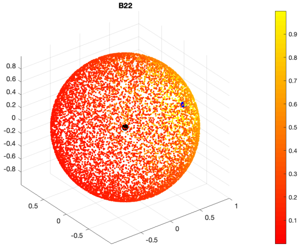
Check the blocks along the diagonal (related to nodes)

- fix n_1 and n_2 while moving n_3 on the sphere

- compute the minimum eigenvalue of the blocks B_{ii}







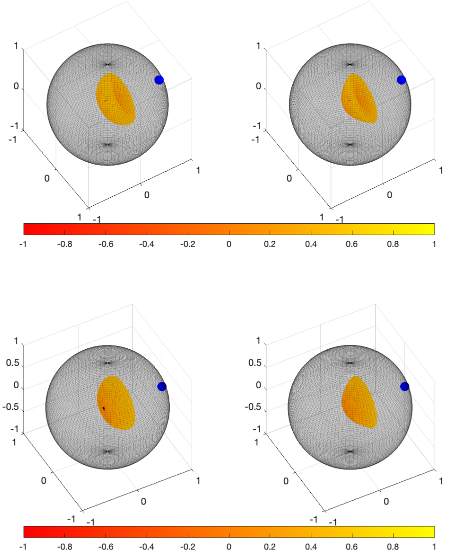
$(6n-7)$ Shades of Rigidity: the 3-agent case

- eigenvalue analysis of the symmetric rigidity matrix...

$\min(\sigma(\mathbf{B}))$ (left)

vs

$\min\{\min(\sigma(B_{ii}))\}$ (right)



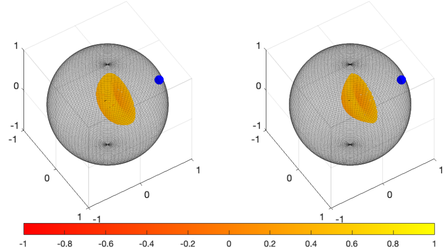
$(6n-7)$ Shades of Rigidity: the 3-agent case

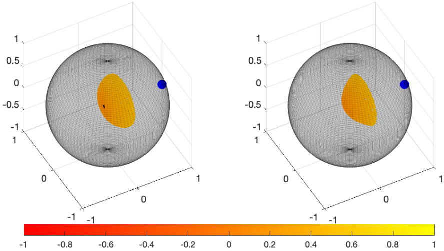
- eigenvalue analysis of the symmetric rigidity matrix...

$$\min(\sigma(\mathbf{B}))\ \text{(left)}$$

vs

$$\min\{\min(\sigma(B_{ii})\}\ \text{(right)}$$





Take-home message: symmetric shapes appear to be "more rigid".

$(6n-7)$ Shades of Rigidity

$$\mathbf{B}_{\mathcal{G}}(\mathbf{x})\mathbf{B}_{\mathcal{G}}(\mathbf{x})^\top = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{bmatrix}$$

- point of view of *edges*...

$$\mathbf{B}_{\mathcal{G}}(\mathbf{x})\mathbf{B}_{\mathcal{G}}^\top(\mathbf{x}) \in \mathbb{R}^{3m\times 3m}$$

$$(1,1) = \mathbf{Q}(d_{12},\mathbf{b}_{12})^2 + [\mathbf{b}_{12}]_\times^2 + \left(\mathbf{Q}(d_{12},\mathbf{b}_{12})\mathbf{R}_1^\top\mathbf{R}_2\right)^2$$

$$(1,2) = \mathbf{Q}(d_{12},\mathbf{b}_{12})\mathbf{Q}(d_{13},\mathbf{b}_{13}) + [\mathbf{b}_{12}]_\times[\mathbf{b}_{13}]_\times$$

$$(2,2) = \mathbf{Q}(d_{13},\mathbf{b}_{13})^2 + [\mathbf{b}_{13}]_\times^2 + \left(\mathbf{Q}(d_{13},\mathbf{b}_{13})\mathbf{R}_1^\top\mathbf{R}_3\right)^2$$

$$(1,3) = -\mathbf{Q}(d_{12},\mathbf{b}_{12})\mathbf{Q}(d_{21},\mathbf{b}_{21})\mathbf{R}_2^\top\mathbf{R}_1 - \mathbf{Q}(d_{12},\mathbf{b}_{12})\mathbf{R}_1^\top\mathbf{R}_2\mathbf{Q}(d_{21},\mathbf{b}_{21})$$

$$(1,4) = -\mathbf{Q}(d_{12},\mathbf{b}_{12})\mathbf{R}_1^\top\mathbf{R}_2\mathbf{Q}(d_{23},\mathbf{b}_{23})$$

$$(2,3) = -\mathbf{Q}(d_{13},\mathbf{b}_{13})\mathbf{Q}(d_{21},\mathbf{b}_{21})\mathbf{R}_2^\top\mathbf{R}_1$$

$$(2,4) = \mathbf{Q}(d_{13},\mathbf{b}_{13})\mathbf{R}_1^\top\mathbf{R}_3\mathbf{Q}(d_{23},\mathbf{b}_{23})\mathbf{R}_2^\top\mathbf{R}_3$$

$(6n-7)$ Shades of Rigidity: the 5-agent case

Noise on edges (measurements)... rigidity-based estimation

Bearing estimation error: $\mathbf{e}(\mathbf{x},\hat{\mathbf{x}}) = \mathbf{b}_{\mathcal{G}}(\mathbf{x}) - \mathbf{b}_{\mathcal{G}}(\hat{\mathbf{x}})$

- estimator

$$\dot{\hat{\mathbf{x}}} = \mathbf{D}(\hat{\mathbf{x}})\mathbf{u}_L = \mathbf{D}(\hat{\mathbf{x}}) \cdot k_e \mathbf{B}_{\mathcal{G}}^{\top}(\hat{\mathbf{x}})\mathbf{b}_{\mathcal{G}}(\mathbf{x})$$

- error dynamics

if $\dot{\mathbf{b}}_{\mathcal{G}}(\mathbf{x}) = 0$

$$\dot{\mathbf{e}}(\mathbf{x},\hat{\mathbf{x}}) = -k_e \mathbf{B}_{\mathcal{G}}(\hat{\mathbf{x}})\mathbf{B}_{\mathcal{G}}^{\top}(\hat{\mathbf{x}})\mathbf{e}(\mathbf{x},\hat{\mathbf{x}})$$

if $\dot{\mathbf{b}}_{\mathcal{G}}(\mathbf{x}) = \mathbf{B}_{\mathcal{G}}(\mathbf{x})\mathbf{u}$

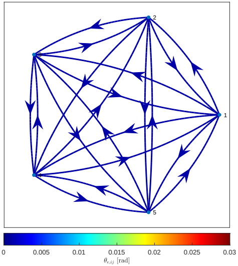
$$\dot{\mathbf{e}}(\mathbf{x},\hat{\mathbf{x}}) = -k_e \mathbf{B}_{\mathcal{G}}(\hat{\mathbf{x}})\mathbf{B}_{\mathcal{G}}^{\top}(\hat{\mathbf{x}})\mathbf{e}(\mathbf{x},\hat{\mathbf{x}}) + (\mathbf{B}_{\mathcal{G}}(\mathbf{x}) - \mathbf{B}_{\mathcal{G}}(\hat{\mathbf{x}}))\mathbf{u}$$

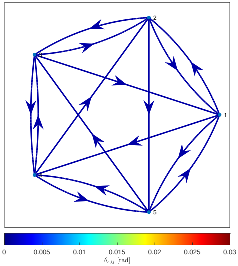
Localization algorithm that estimates the state $\mathbf{x}$ of the formation.
The initial formation is static ($\dot{\mathbf{b}}_{\mathcal{G}}(\mathbf{x}) = 0$).
The estimator is initialized by perturbing the position and orientation of the agents.
Estimation performance is evaluated in a fixed interval.

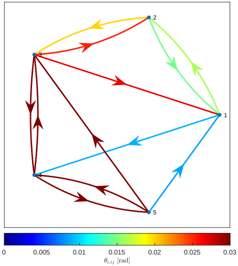
$(6n-7)$ Shades of Rigidity: the 5-agent case

Noiseless scenario. Graph: complete → -5 edges → -8 edges.
Bearing error along edges: $\theta_{e,ij} = \arccos(\hat{\mathbf{b}}_{ij} \bullet \mathbf{b}_{ij})$

- λ_8 decreases (rigidity is preserved): $0.88 \rightarrow 0.63 \rightarrow 0.26$
- $\|\mathbf{B}_{\mathcal{G}}(\mathbf{x})\|_F$ decreases: $8.94 \rightarrow 8.03 \rightarrow 7.31$
- estimation performance decreases


Bearing errors. Complete graph. λ_{min}=0.880486


Bearing errors. Non-complete graph. λ_{min}=0.633362

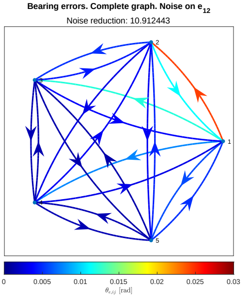

Bearing errors. Non-complete graph. λ_{min}=0.256742

$(6n-7)$ Shades of Rigidity: the 5-agent case

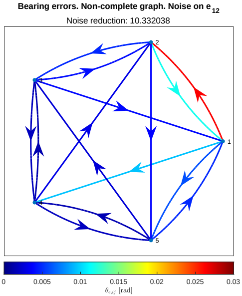
Noisy scenario ($\mathbf{e}_{12}$). Graph: complete → −5 edges → −8 edges.
Bearing error along edges: $\theta_{e,ij} = \arccos(\hat{\mathbf{b}}_{ij} \bullet \mathbf{b}_{ij})$

- noise reduction decreases: 10.91 → 10.33 → 9.16
- pattern of noise propagation is not trivial

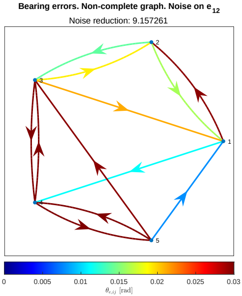
Take-home message: noise modifies the rigidity matrix.


Bearing errors. Complete graph. Noise on e_{12}

Noise reduction: 10.912443


Bearing errors. Non-complete graph. Noise on e_{12}

Noise reduction: 10.332038


Bearing errors. Non-complete graph. Noise on e_{12}

Noise reduction: 9.157261

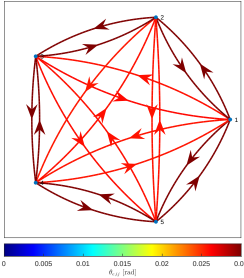
$(6n-7)$ Shades of Rigidity: the 5-agent case

Noisy scenario ($\mathbf{e}_\forall$). Graph: complete → -5 edges → -8 edges.
Bearing error along edges: $\theta_{e,ij} = \arccos(\hat{\mathbf{b}}_{ij} \bullet \mathbf{b}_{ij})$

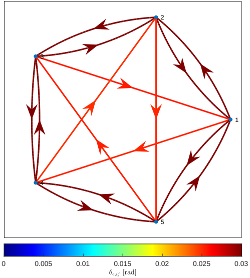
- in symmetric structures noise reduction is bi-modal: ≈ 10 vs ≈ 8
  "distant nodes" behave better (state estimation)

> *Take-home message*: distances play a role also when it comes to bearings.

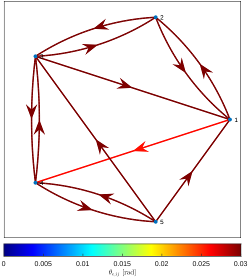
Bearing errors. Complete graph. Noise on all.



Bearing errors. Non-complete graph. Noise on all.



Bearing errors. Non-complete graph. Noise on all.



$(6n-7)$ Shades of Rigidity: the 5-agent case

Noise on edges (measurements)... spectral effects

		complete	-5 edges	-8 edges
Noiseless scenario	rank $\mathbf{B}_{\mathcal{G}}(\mathbf{x})$	23	23	23
	σ_8	0.88	0.63	0.26
	$\Vert\mathbf{B}_{\mathcal{G}}(\mathbf{x})\Vert_F$	8.94	8.03	7.30
Noisy scenario $(\mathbf{e}_{12})$	rank $\mathbf{B}_{\mathcal{G}}(\hat{\mathbf{x}})$	23	23	23
	σ_8	0.79 ± 0.08	0.52 ± 0.05	0.19 ± 0.04
	$\Vert\mathbf{B}_{\mathcal{G}}(\hat{\mathbf{x}})\Vert_F$	9.40 ± 1.22	8.06 ± 0.63	7.45 ± 0.87
Noisy scenario $(\mathbf{e}_{\forall})$	rank $\mathbf{B}_{\mathcal{G}}(\hat{\mathbf{x}})$	23	23	23
	σ_8	0.78 ± 0.09	0.53 ± 0.05	0.16 ± 0.05
	$\Vert\mathbf{B}_{\mathcal{G}}(\hat{\mathbf{x}})\Vert_F$	9.15 ± 0.89	8.19 ± 1.18	7.47 ± 0.61

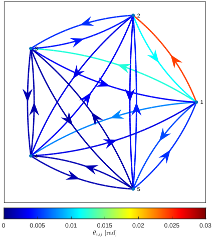
Take-home message: noise modifies the rigidity matrix.

($6n-7$) Shades of Rigidity: the 5-agent case

Noise on edges (measurements)... both bearings and distances

Bearing errors. Complete graph. Noise on e_{12}

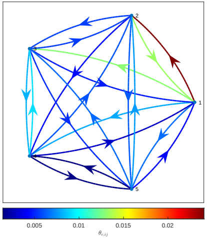
Noise reduction: 10.912443



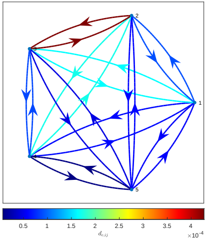
$\rightsquigarrow$

Bearing errors. Complete graph. B-noise on e_{12} + D-noise on e_{32}

B-noise reduction: 11.040482



D-noise reduction 18.621262



Take-home message: distances play a role also when it comes to bearings.

Conclusion

In a nutshell...

- rigidity can be interpreted **beyond the binary feature** through spectral analysis: this is affected by both structural and scenario properties

- bearing rigidity properties of a multi-agent system depend on formation **scaling** and on **measurements homogeneity**: this affects formation control and estimation capabilities

- the **presence of noise** affects the performance of both estimate and control tasks since it modifies **the bearing rigidity matrix**

- noise action is **not structural** (sparsity is preserved) but **numerical** (consistency is perturbed)

...there are more things in heaven and earth, Horatio,
than are dreamt of in your philosophy

Hamlet, Act I - Scene 5

Thank you for your attention

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