Cooperative Manipulation via Internal-Force Regulation: A Rigidity Theory Perspective

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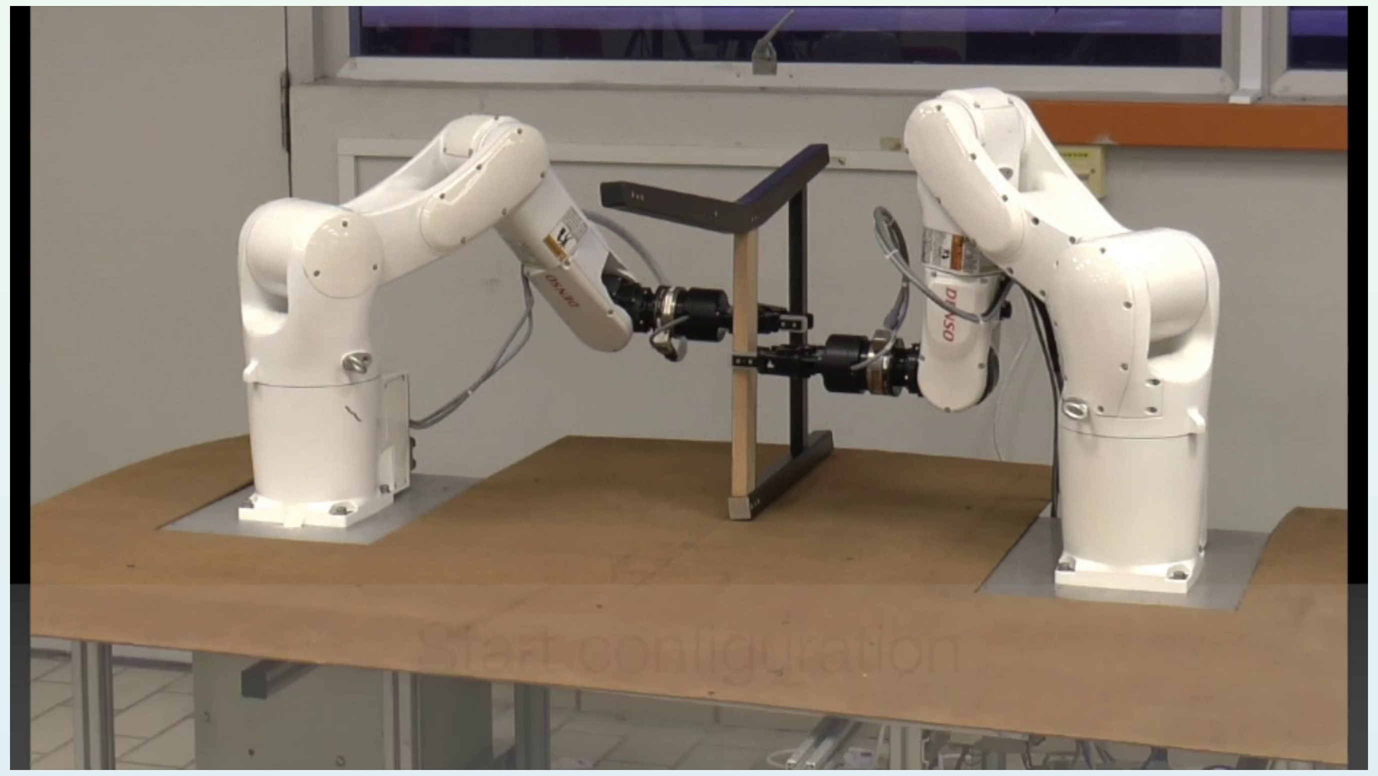
Motivation

Cooperative manipulation

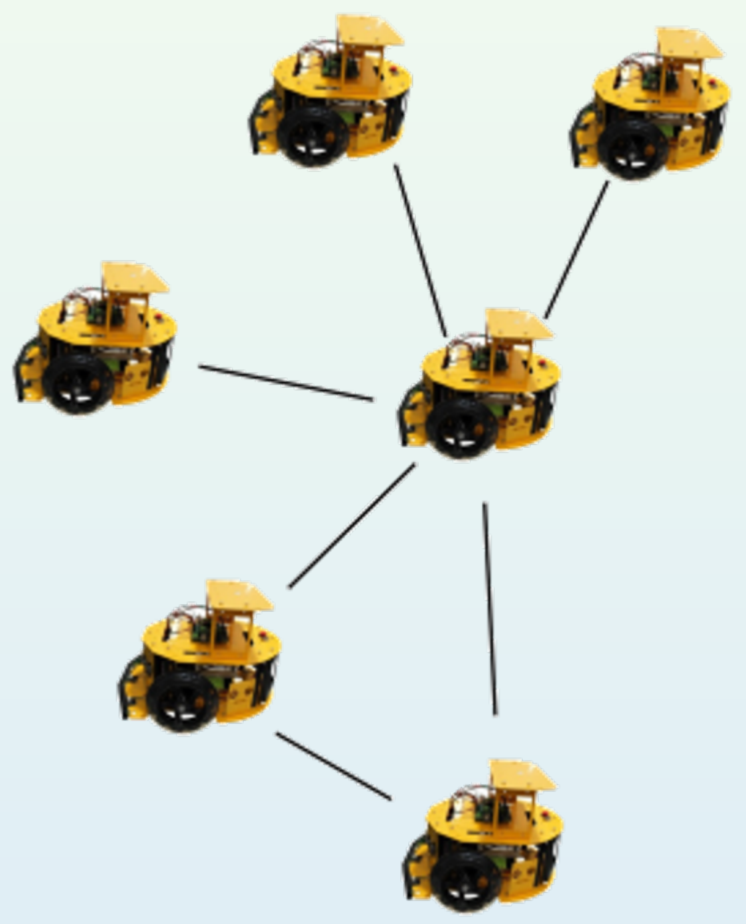
- Large/Heavy payloads
- Challenging manoeuvres
- Bimanual tasks

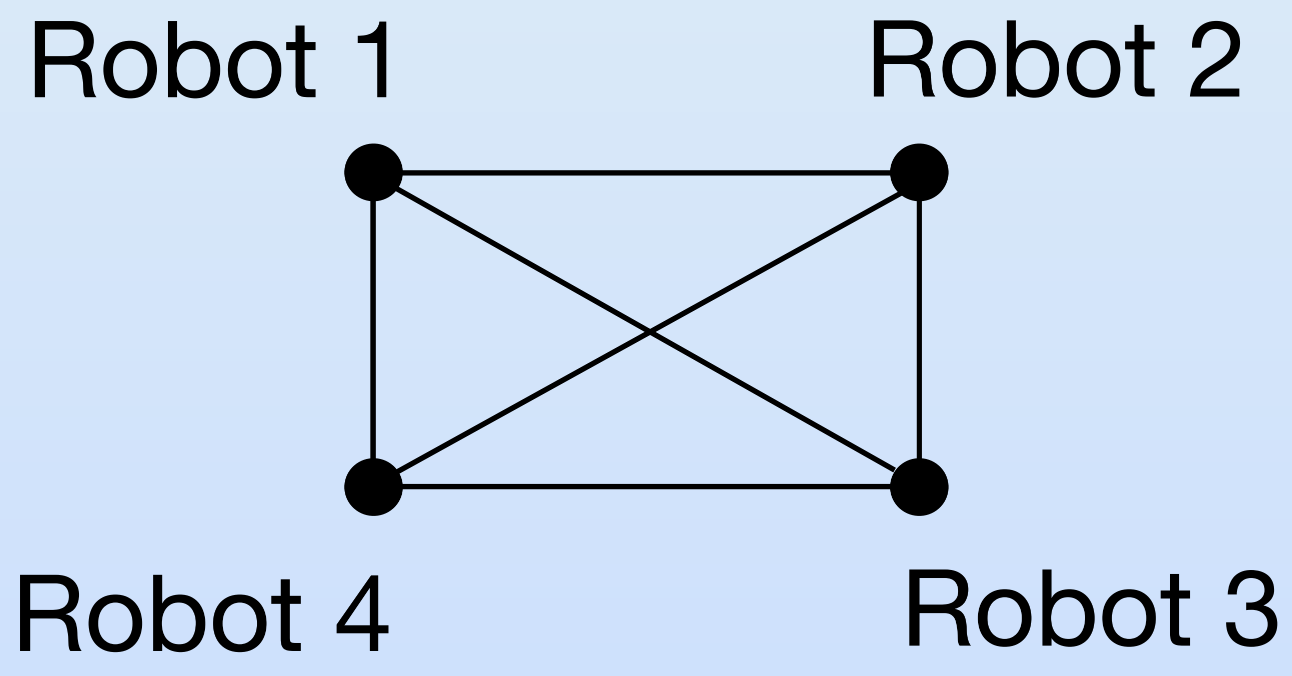


Motivation

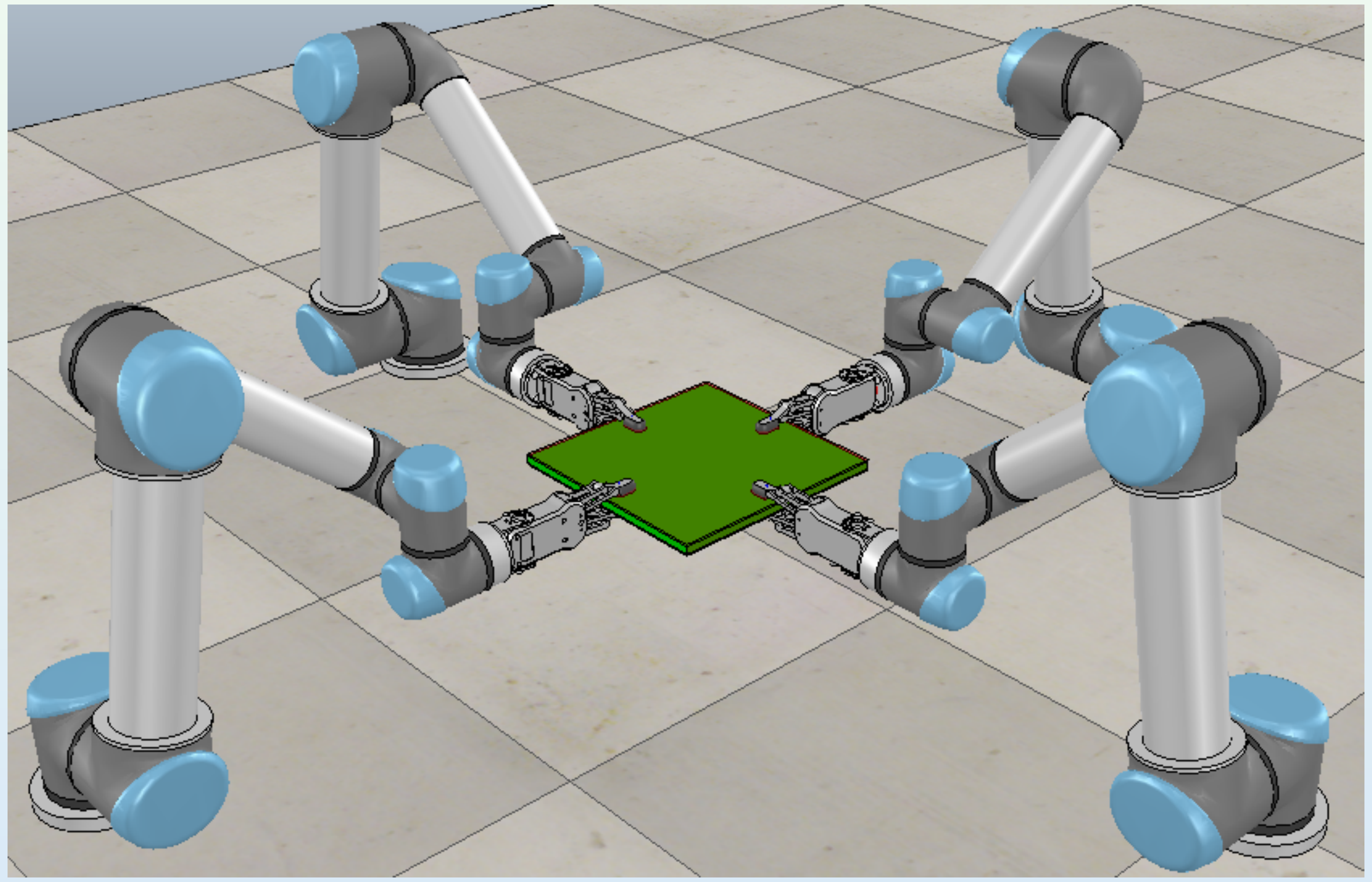
Rigidity theory

- Understand relations in multi-agent networks
- Formation control
- Distance rigidity, bearing/angle rigidity





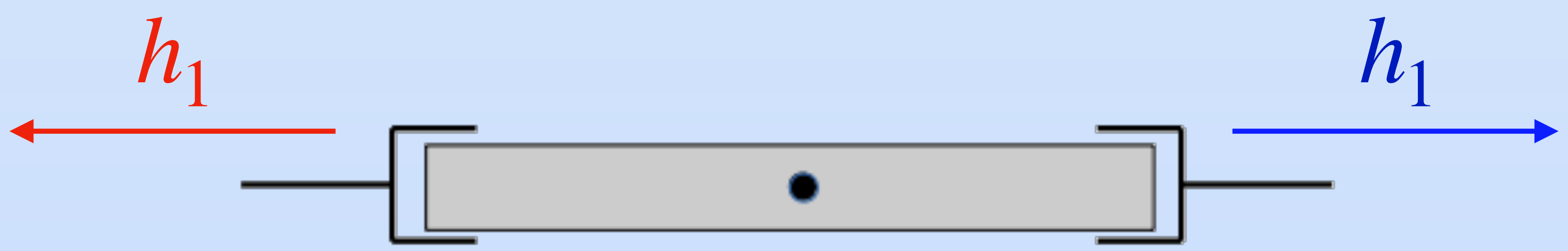
Motivation



Rigid cooperative manipulation

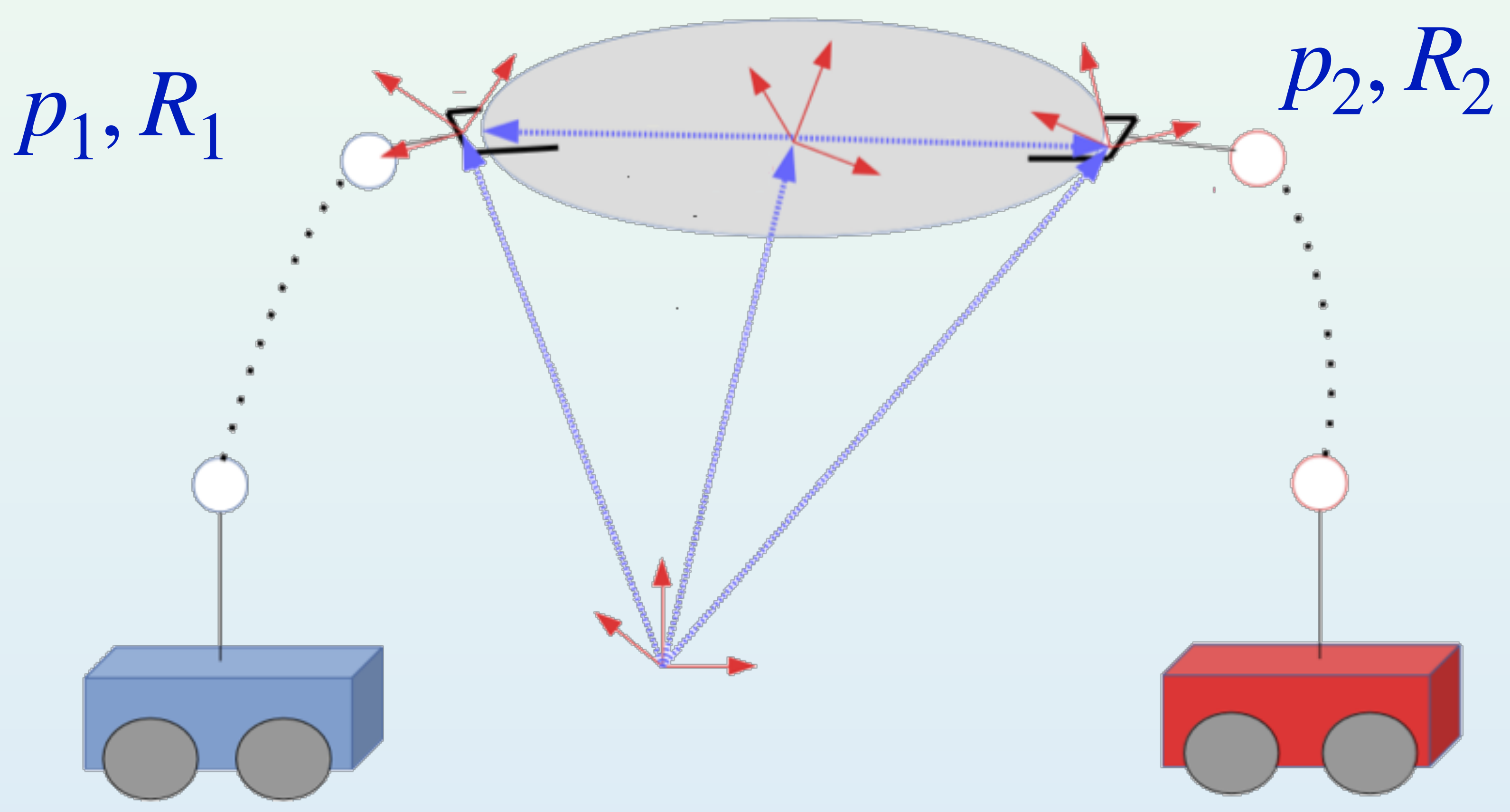
- A group of robots grasp an object *rigidly*
- Aim to achieve trajectory tracking by object's COM
- How do we *minimize* internal forces?

Internal forces: forces produced by the agents that do not contribute to the object's motion



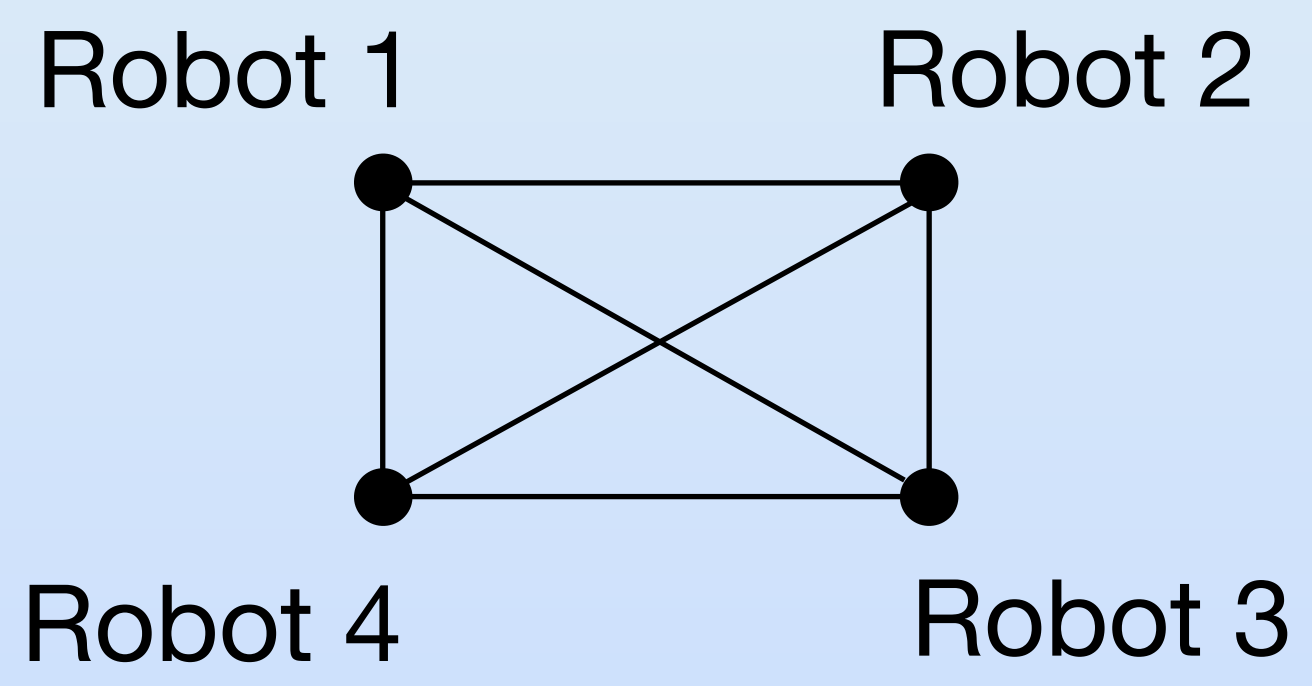
Distance and bearing rigidity in $SE(3)$

- Characterise motions that preserve *distances* and *bearings* between agents $\mathcal{N} = \{1, \ldots, N\}$



- Multi-agent network:
    - Directed graph: $\mathcal{E} \subseteq \{(i,j) \in \mathcal{N} : i \neq j\}$
    - Undirected graph: $\mathcal{E}_u = \{(i,j) \in \mathcal{E} : i < j\}$

- Distance and bearing constraints:
    - Distance: $\gamma_{e,d} = \frac{1}{2}\|p_i - p_j\|^2,\ e \in \mathcal{E}_u$
    - Bearing: $\gamma_{e,b} = R_i^\top \frac{p_i - p_j}{\|p_i - p_j\|},\ e \in \mathcal{E}$
    - D&B function: $\gamma_{\mathcal{G}} = \left[\gamma_{1,d}, \ldots, \gamma_{|\mathcal{E}_u|,d}, \gamma_{1,b}^\top, \ldots, \gamma_{1,|\mathcal{E}|}^\top\right]^\top$

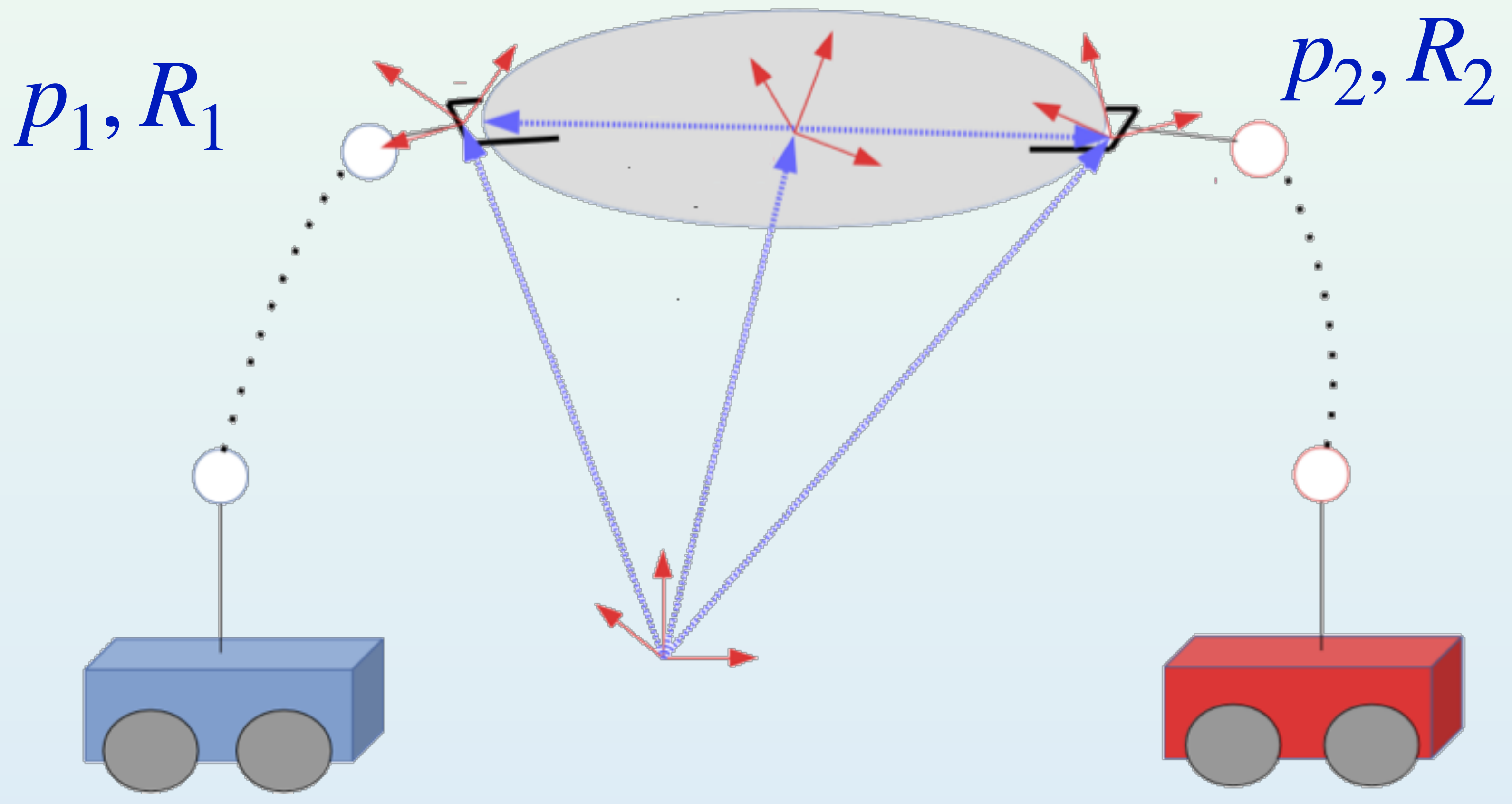


Infinitesimal distance and bearing rigidity in SE(3)

Infinitesimal motions: motion-perturbations of the agents that leave the rigidity function $\gamma_{\mathcal{G}}$ unchanged

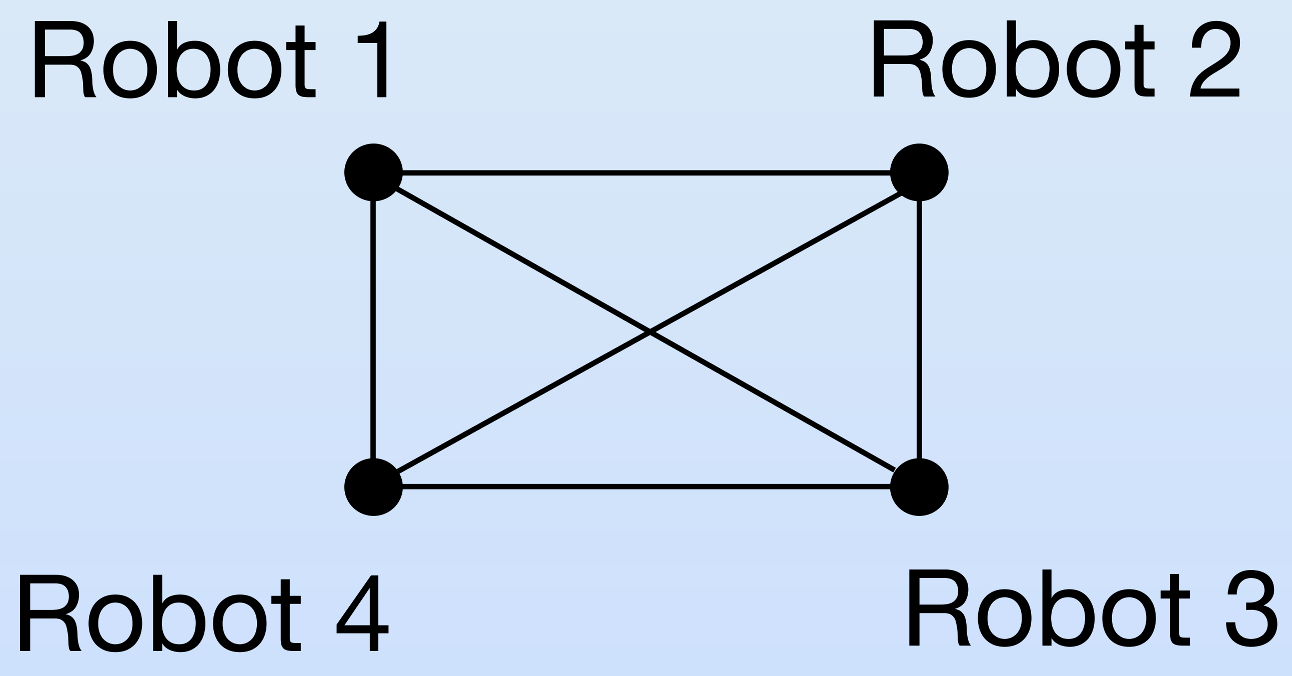
Rigidity matrix

$$\mathcal{R}_{\mathcal{G}}(x) = \left[\frac{\partial \gamma_{\mathcal{G}}}{\partial p_1}, \frac{\partial \gamma_{\mathcal{G}}}{\partial R_1}, \ldots, \frac{\partial \gamma_{\mathcal{G}}}{\partial p_N}, \frac{\partial \gamma_{\mathcal{G}}}{\partial R_N}\right]$$



Infinitesimal motions: motions $x(t)$ produced by the nullspace of $\mathcal{R}_{\mathcal{G}}(x)$, i.e.,

$$\dot{\gamma}_{\mathcal{G}} = \mathcal{R}_{\mathcal{G}}(x(t))\dot{x}(t) = 0$$



Trivial motions: motions that preserve the distances and bearings of the system

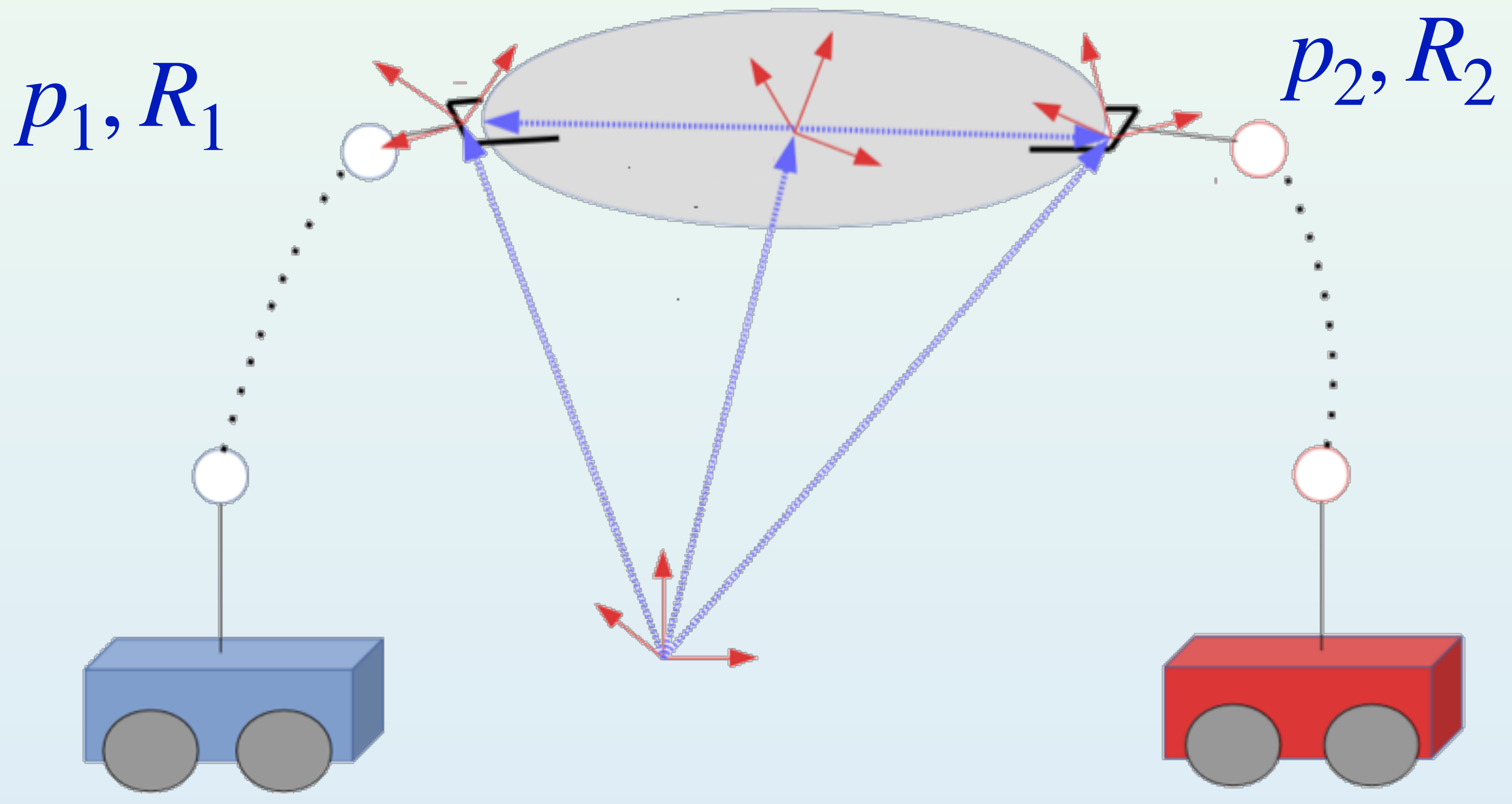
D&B infinitesimally rigid system:
infinitesimal motions are trivial motions

Infinitesimal distance and bearing rigidity in $SE(3)$

Infinitesimal motions: motion-perturbations of the agents that leave the rigidity function $\gamma_{\mathcal{G}}$ unchanged

Rigidity matrix

$$\mathcal{R}_{\mathcal{G}}(x) = \left[\frac{\partial \gamma_{\mathcal{G}}}{\partial p_1}, \frac{\partial \gamma_{\mathcal{G}}}{\partial R_1}, \ldots, \frac{\partial \gamma_{\mathcal{G}}}{\partial p_N}, \frac{\partial \gamma_{\mathcal{G}}}{\partial R_N}\right]$$



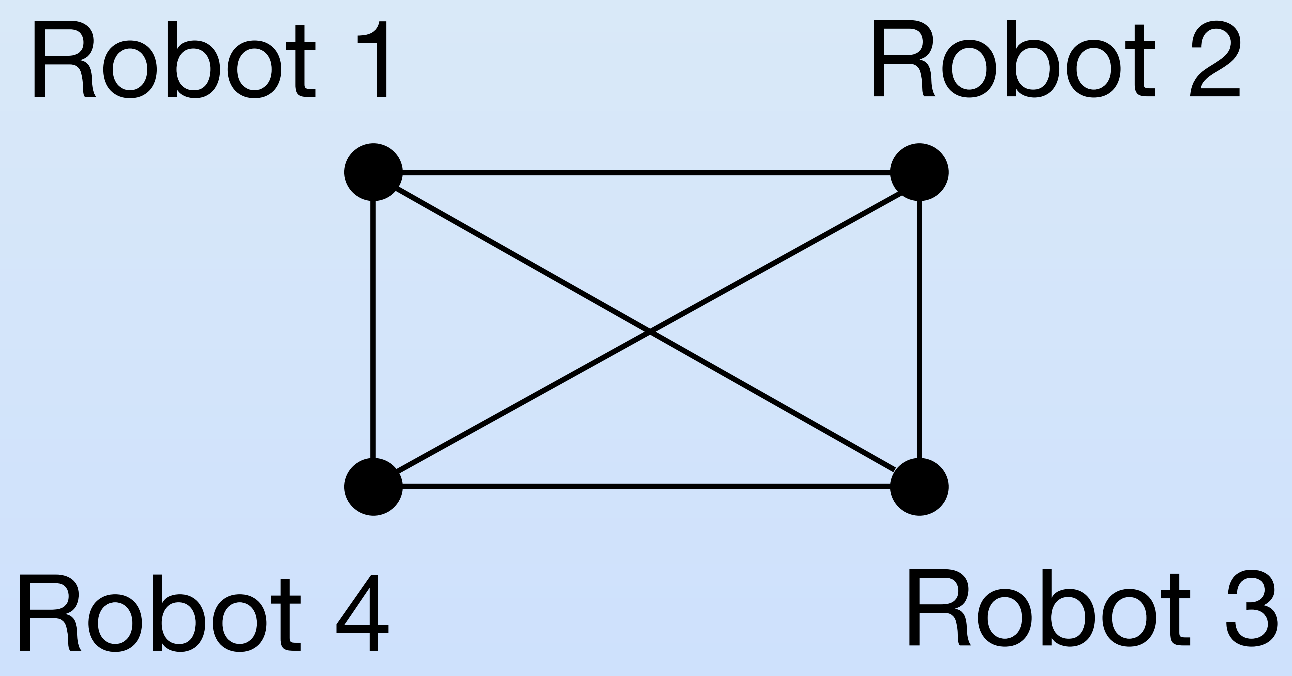
The multi-agent system is D&B infinitesimally rigid in $SE(3)$ if and only if

$$\text{rank}\left(\widetilde{\mathcal{R}}_{\mathcal{G}}\right) = 6N - 6$$

where

$$\widetilde{\mathcal{R}}_{\mathcal{G}}(x) = \left[\frac{\partial \gamma_{\mathcal{G}}}{\partial p_1}, \ldots, \frac{\partial \gamma_{\mathcal{G}}}{\partial p_N}, \frac{\partial \gamma_{\mathcal{G}}}{\partial R_1}, \ldots, \frac{\partial \gamma_{\mathcal{G}}}{\partial R_N}\right]$$

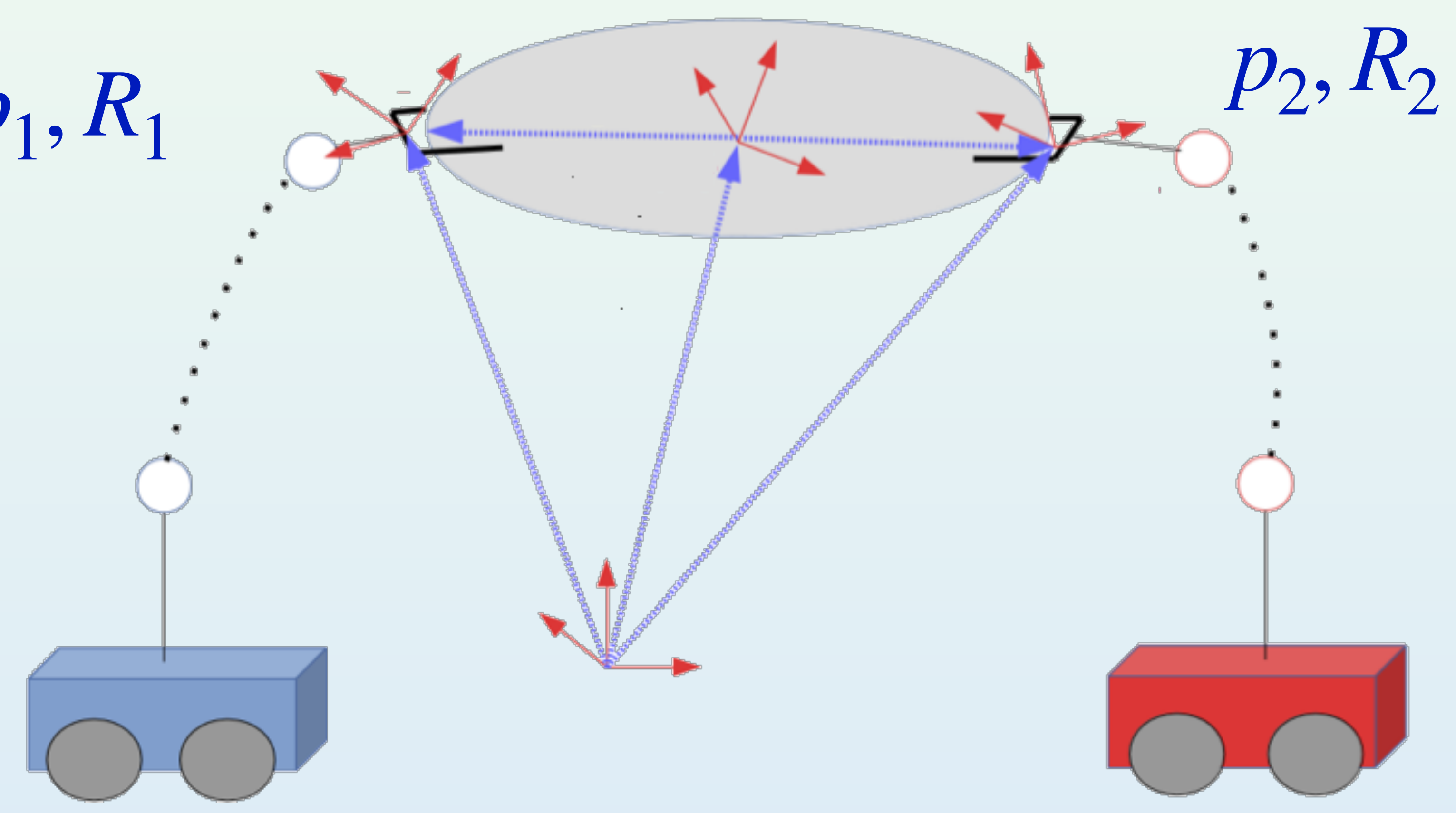
is a column-permutation of $\mathcal{R}_{\mathcal{G}}$



Cooperative manipulation modelling

- Robot dynamics $\begin{cases} v = [\dot{p}_1^\top, \omega_1^\top, \ldots, \dot{p}_N^\top, \omega_N^\top]^\top \\ \dot{R}_i = S(\omega_i)R_i \\ M(x)\dot{v} + C(x,\dot{x})v + g(x) = u - h \end{cases}$

Skew-symmetric matrix



- Object dynamics $\begin{cases} v_o = [\dot{p}_o^\top, \omega_o^\top]^\top \\ \dot{R}_o = S(\omega_o)R_o \\ M_o(x)\dot{v}_o + C(x_o,\dot{x}_o)v_o + g_o(x) = h_o \end{cases}$

- Object-robots coupling:
  - Velocities: $v = G(x)^\top v_o$
  - Forces: $h_o = G(x)h$

Grasp matrix: $G(x) = \left[J_{o_1}(x_1)^\top, \ldots, J_{o_N}(x_N)^\top\right] \in \mathbb{R}^{6\times 6N}$

$$J_{o_i}(x_i) = \begin{bmatrix} I_3 & -S(p_i - p_o) \\ 0_{3\times 3} & I_3 \end{bmatrix}, i \in \mathcal{N}$$

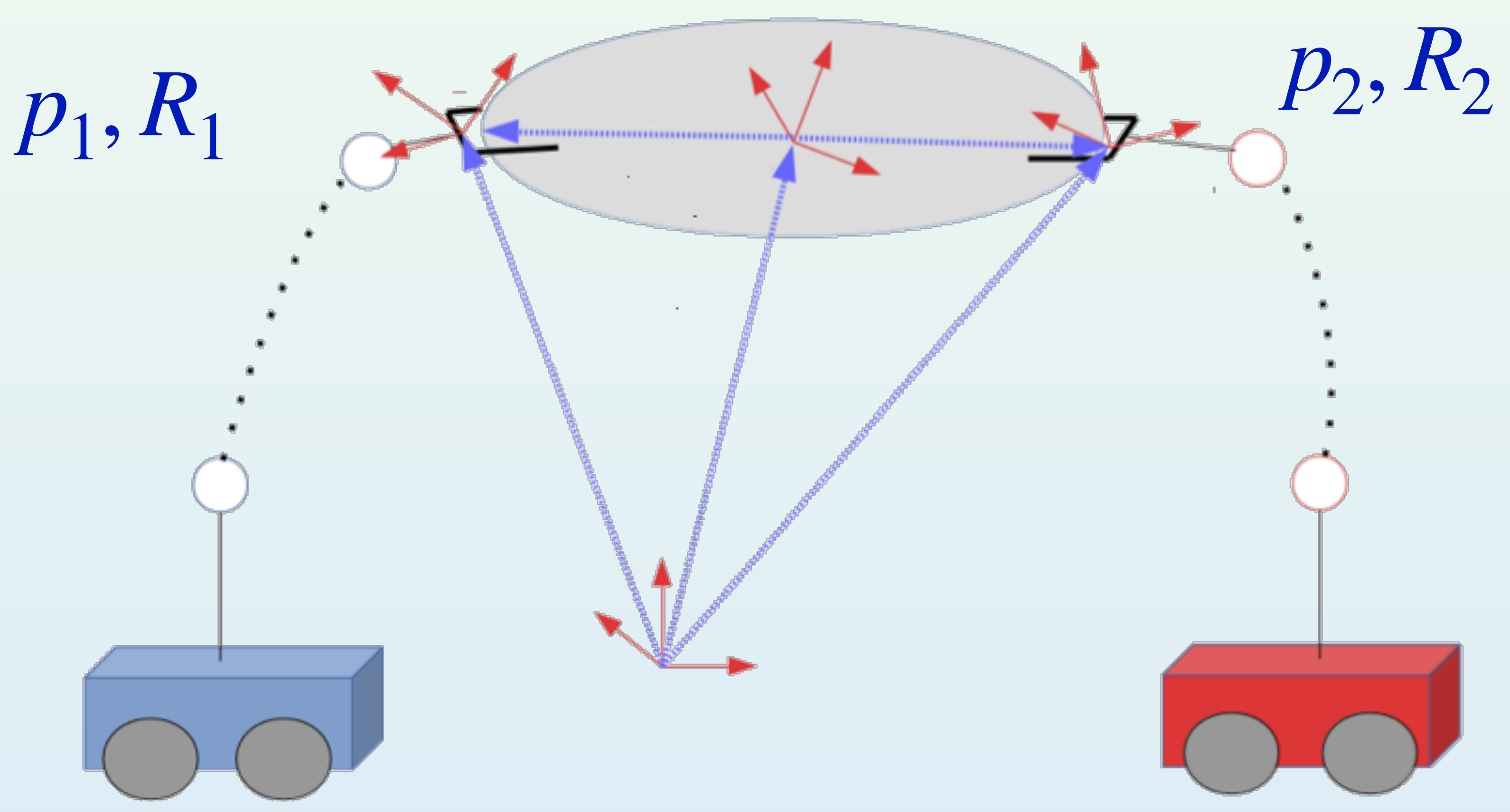
- Force decomposition: $h = h_m + h_{int}$

motion-inducing forces

internal forces $(G(x)h_{int} = 0)$

Internal forces based on the D&B rigidity matrix

- View the cooperative manipulation as a graph
  - Interactions among all agents ⟶ *complete graph*



Distance and bearing rigidity in $\mathrm{SE}(3)$

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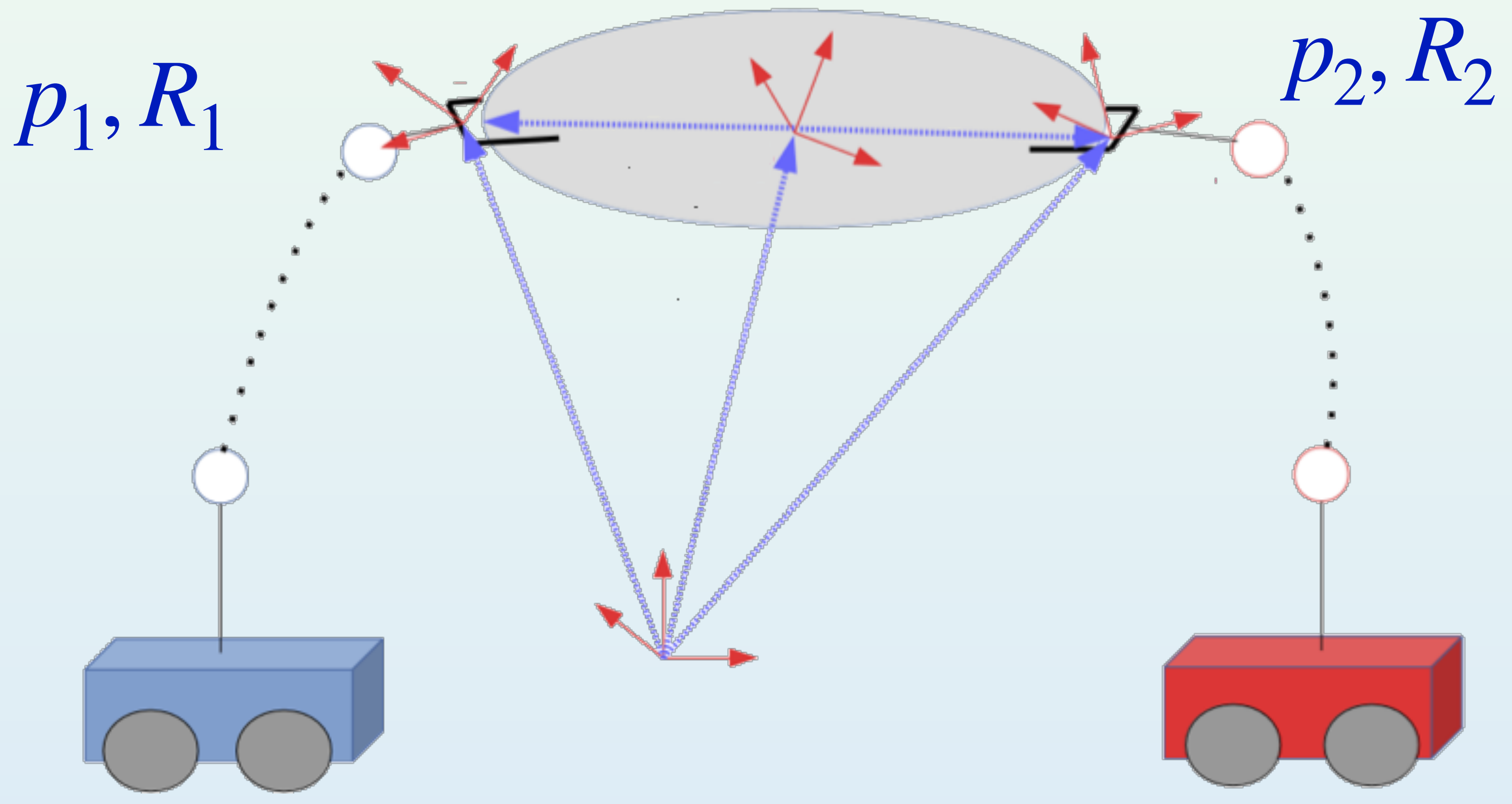

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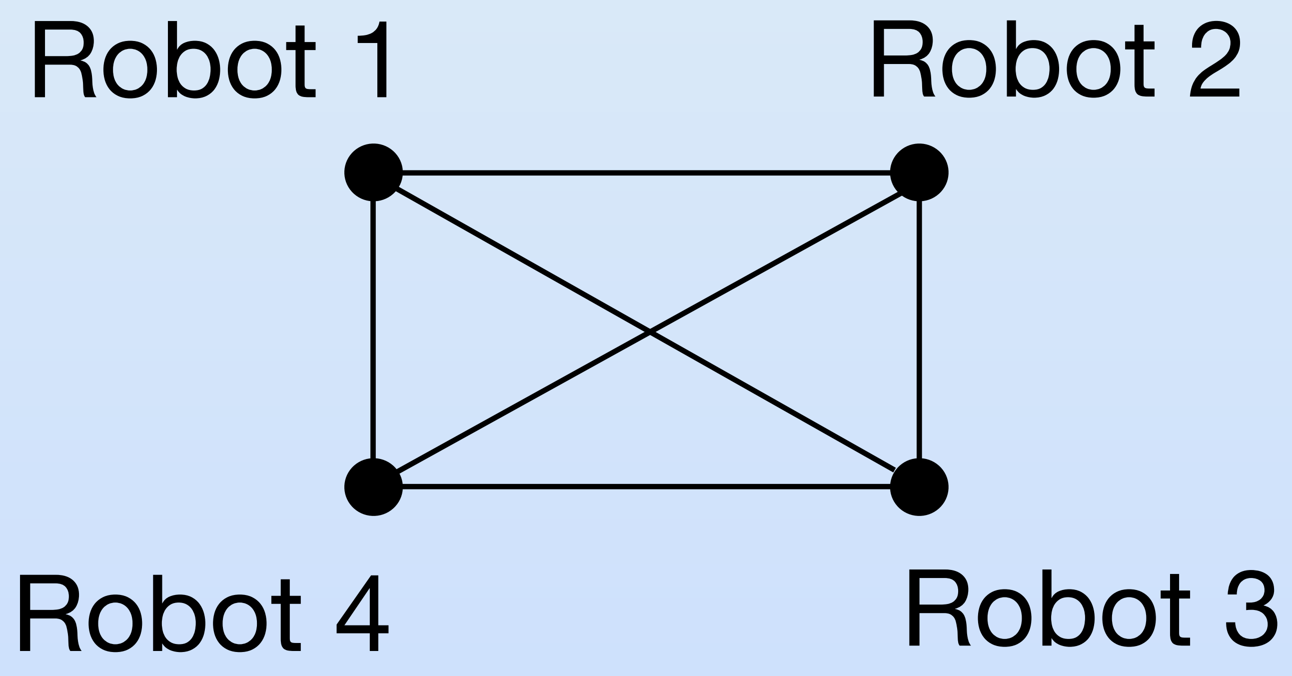
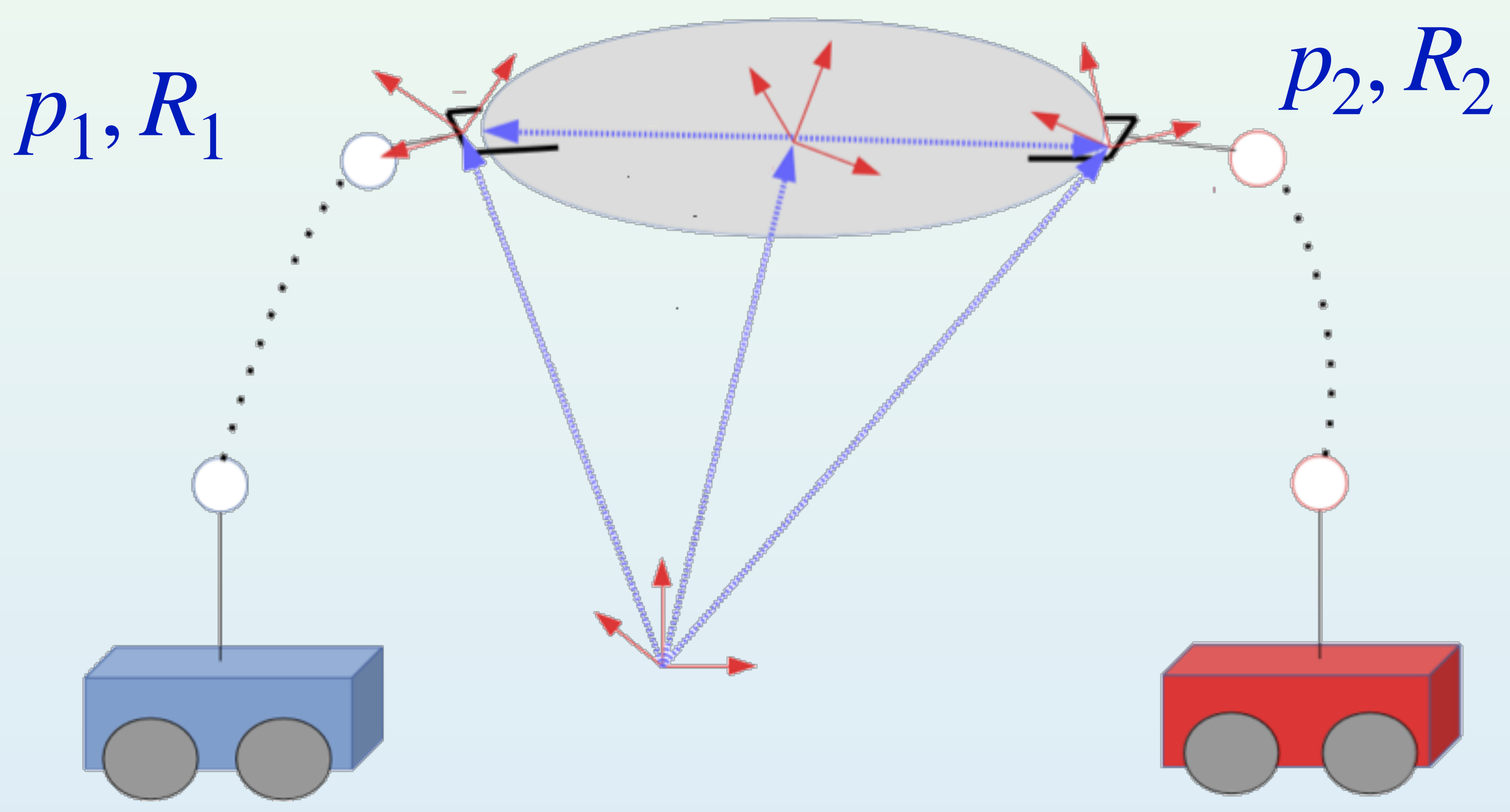


- Distance and bearing constraints:
    - Distance: $\gamma_{e,d} = \frac{1}{2}\|p_i - p_j\|^2,\ e \in \mathcal{E}_u$
    - Bearing: $\gamma_{e,b} = R_i^\top \frac{p_i - p_j}{\|p_i - p_j\|},\ e \in \mathcal{E}$
    - D&B function: $\gamma_{\mathcal{G}} = \left[\gamma_{1,d}, \ldots, \gamma_{|\mathcal{E}_u|,d}, \gamma_{1,b}^\top, \ldots, \gamma_{1,|\mathcal{E}|}^\top\right]^\top$

Internal forces based on the D&B rigidity matrix

- View the cooperative manipulation as a graph
  - Interactions among all agents $\longrightarrow$ *complete graph*



- Cooperative manipulation rigidity constraints: $\gamma_{\mathcal{G}} = 0$

$$\Rightarrow \quad \mathcal{R}_{\mathcal{G}} \dot{v} = -\dot{\mathcal{R}}_{\mathcal{G}}(x) v$$

- Complete graph: $\mathcal{R}_{\mathcal{G}}$ encodes rigid body motions

- Internal-force dynamics: $M(x)\dot{v} + C(x, \dot{x})v + g(x) = u - h_{int}$
- Unconstrained dynamics: $M(x)\alpha + C(x, \dot{x})v + g(x) = u$

- Gauss' principle:

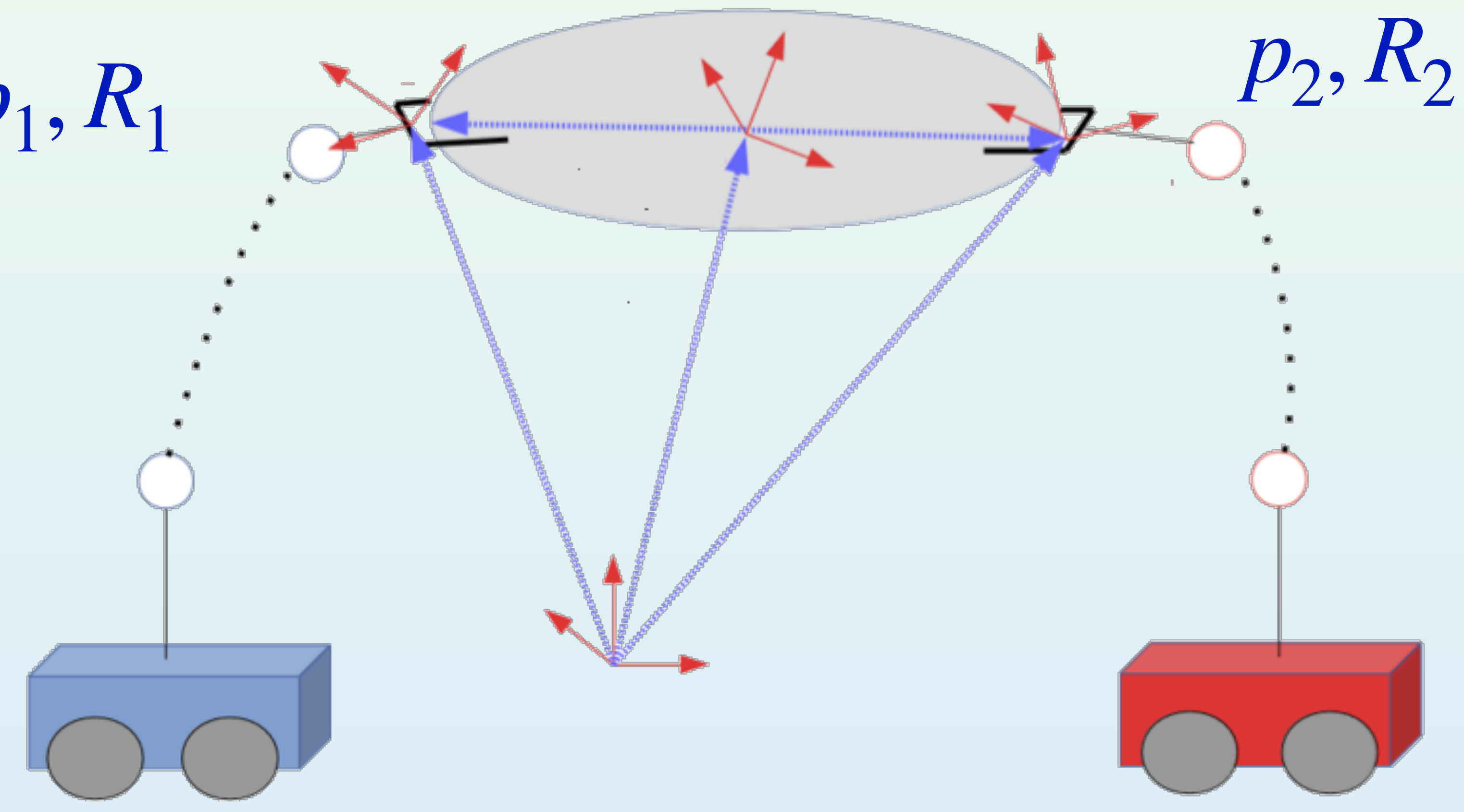
$$\boxed{\begin{aligned} \dot{v} &= \min\ (\dot{v} - a)^\top M(x)\ (\dot{v} - a) \\ &\text{s.t.}\ \ \mathcal{R}_{\mathcal{G}} \dot{v} = -\dot{\mathcal{R}}_{\mathcal{G}} v \end{aligned}} \longrightarrow$$

Internal forces:

$$\boxed{h_{int} = M^{\frac{1}{2}} \left(\mathcal{R}_{\mathcal{G}} M^{-\frac{1}{2}} \right) (\dot{\mathcal{R}}_{\mathcal{G}} v + \mathcal{R}_{\mathcal{G}} a)}$$

Internal forces based on the D&B rigidity matrix

Internal forces: $\begin{cases} h_{int} = M^{\frac{1}{2}}\left(\mathscr{R}_{\mathscr{G}} M^{-\frac{1}{2}}\right)(\dot{\mathscr{R}}_{\mathscr{G}} v + \mathscr{R}_{\mathscr{G}} a) \\ \alpha = M(x)^{-1}(u - C(x, \dot{x})v + g(x)) \end{cases}$



Let $\mathscr{R}_{\mathscr{G},1}$ and $\mathscr{R}_{\mathscr{G},2}$ such that $\text{null}(\mathscr{R}_{\mathscr{G},1}) = \text{null}(\mathscr{R}_{\mathscr{G},2})$ and let

$$h_{int,i} = M^{\frac{1}{2}}\left(\mathscr{R}_{\mathscr{G},i} M^{-\frac{1}{2}}\right)(\dot{\mathscr{R}}_{\mathscr{G},i} v + \mathscr{R}_{\mathscr{G},i} a), \quad i \in \{1,2\}$$

Then $h_{int,1} = h_{int,2}$

The cooperative manipulation system is free of internal forces if and only if

$$\dot{\mathscr{R}}_{\mathscr{G}} v + \mathscr{R}_{\mathscr{G}} M^{-1}(u - C\dot{v} - g) \in \text{null}(\mathscr{R}_{\mathscr{G}}^{\top})$$

Cooperative manipulation via internal-force regulation

Association of $G(x)$ and $\mathscr{R}_{\mathscr{G}}(x)$

Theorem 1.
Let N agents rigidly grasping an object, associated with a grasp matrix $G(x)$. Let also the agents be modelled by a complete graph associated with a rigidity matrix $\mathscr{R}_{\mathscr{G}}(x)$
Then

$$\mathrm{null}(G(x)) = \mathrm{range}(\mathscr{R}_{\mathscr{G}}(x)^\top)$$

$$h_o = G(x)h$$
$$v = G(x)^\top v_o$$

$$\mathscr{R}_{\mathscr{G}}(x) = \left[\frac{\partial \gamma_{\mathscr{G}}}{\partial p_1}, \frac{\partial \gamma_{\mathscr{G}}}{\partial R_1}, \ldots, \frac{\partial \gamma_{\mathscr{G}}}{\partial p_N}, \frac{\partial \gamma_{\mathscr{G}}}{\partial R_N}\right]$$

Cooperative manipulation via internal-force regulation

Relation between forces h and internal forces h_{int}

Theorem 2.
Let N agents rigidly grasping an object. The agent internal forces h_{int} and forces h are related via

$$h_{int} = \left(I_{6N} - MG^\top (GMG^\top)^{-1} G\right) h$$

As opposed to $h_{int} = \left(I_{6N} - G^\top (GG^\top)^{-1} G\right) h$ found in the literature

Cooperative manipulation via internal-force regulation

Internal-force free distribution of a force to the agents

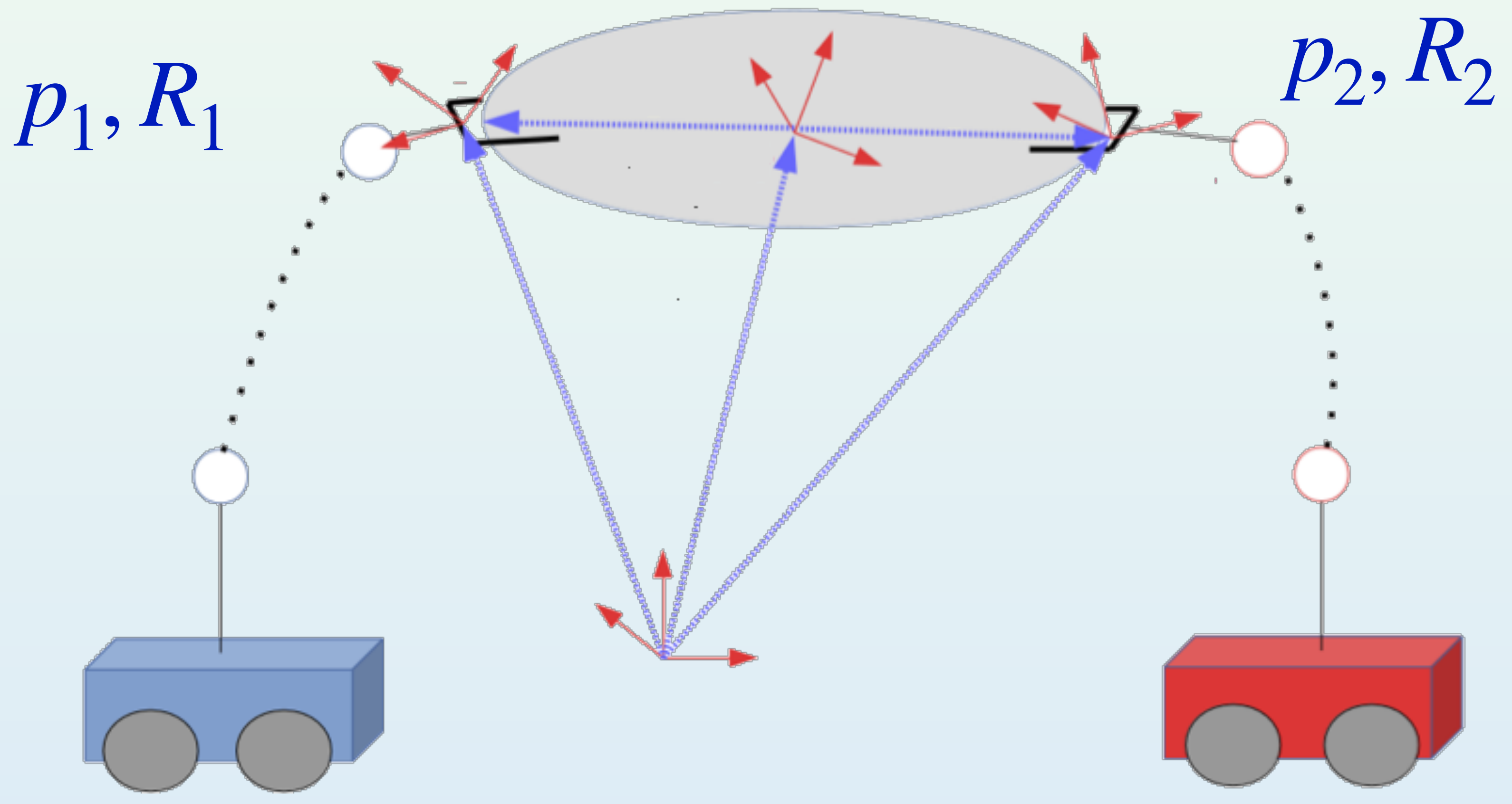
Theorem 3.
Let N agents rigidly grasping an object. Let a desired force $h_{o,d}$ to be applied to the object, distributed to the agents via $h_d = G^* h_{o,d}$, where G^* is a right inverse of G, i.e., $GG^* = I_6$
Then

$$h_{int} = 0 \Leftrightarrow G^* = MG^\top (GMG^\top)^{-1}$$

$h_o = G(x)h$

Control design

- Tracking of reference trajectory $\begin{cases} v_d = [\dot{p}_d^\top, \omega_d^\top]^\top \\ \dot{R}_d = S(\omega_d)R_d \end{cases}$



- Associated errors $\begin{cases} e_p = p_o - p_d \\ e_o = \frac{1}{2}\mathrm{tr}\left(I_3 - R_d^\top R_o\right) \\ e_R = S^{-1}(R_d^\top R_o - R_o^\top R_d) \\ e_x = \left[e_p^\top, \frac{1}{2(2-e_o)^2} e_R^\top R_o^\top\right]^\top \\ e_v = v_o - v_d \end{cases}$

- Internal-force-free control law:

$$
\boxed{\begin{aligned} u = g + (CG^\top + M\dot{G}^\top)v_o + G^*(g_o + C_o v_o) \\ +(MG^\top + G^* M_o)(\dot{v}_d - K_d e_v - K_p e_x) \\ h_{int} = 0 \Leftrightarrow G^* = MG^\top(GMG^\top)^{-1} \end{aligned}}
$$

Simulations

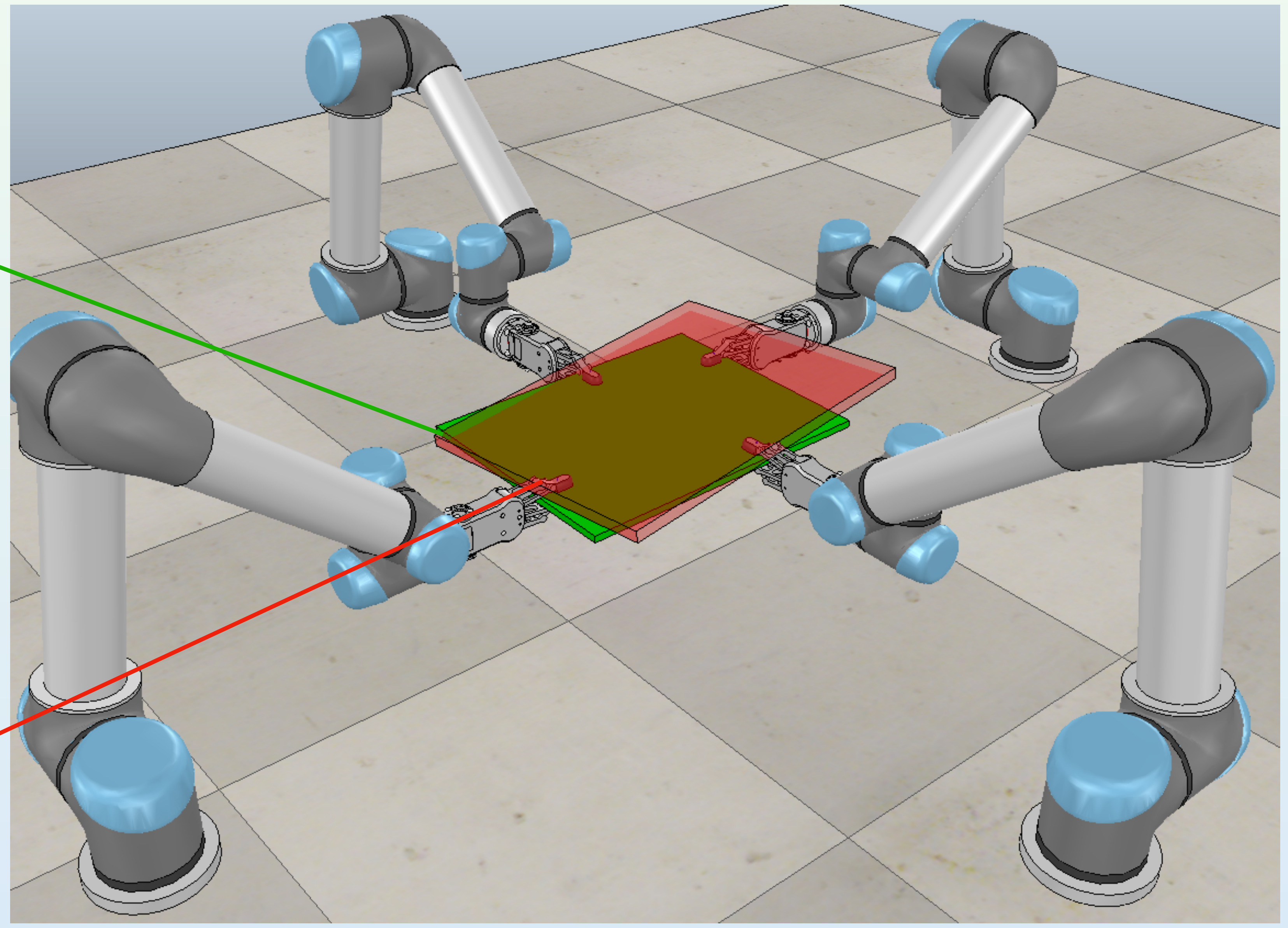
- Tracking of reference trajectory

$$p_d(t) = p_o(0) + \begin{bmatrix} 0.2\sin(t+\frac{\pi}{6}) \\ 0.2\cos(t+\frac{\pi}{6}) \\ 0.09 + 0.1\sin(t+\frac{\pi}{6}) \end{bmatrix}$$

$$\eta_d(t) = 0.15 \begin{bmatrix} \sin(t+\frac{\pi}{6}) \\ \sin(0.5t+\frac{\pi}{6}) \\ \sin(t+\frac{\pi}{6}) \end{bmatrix}$$

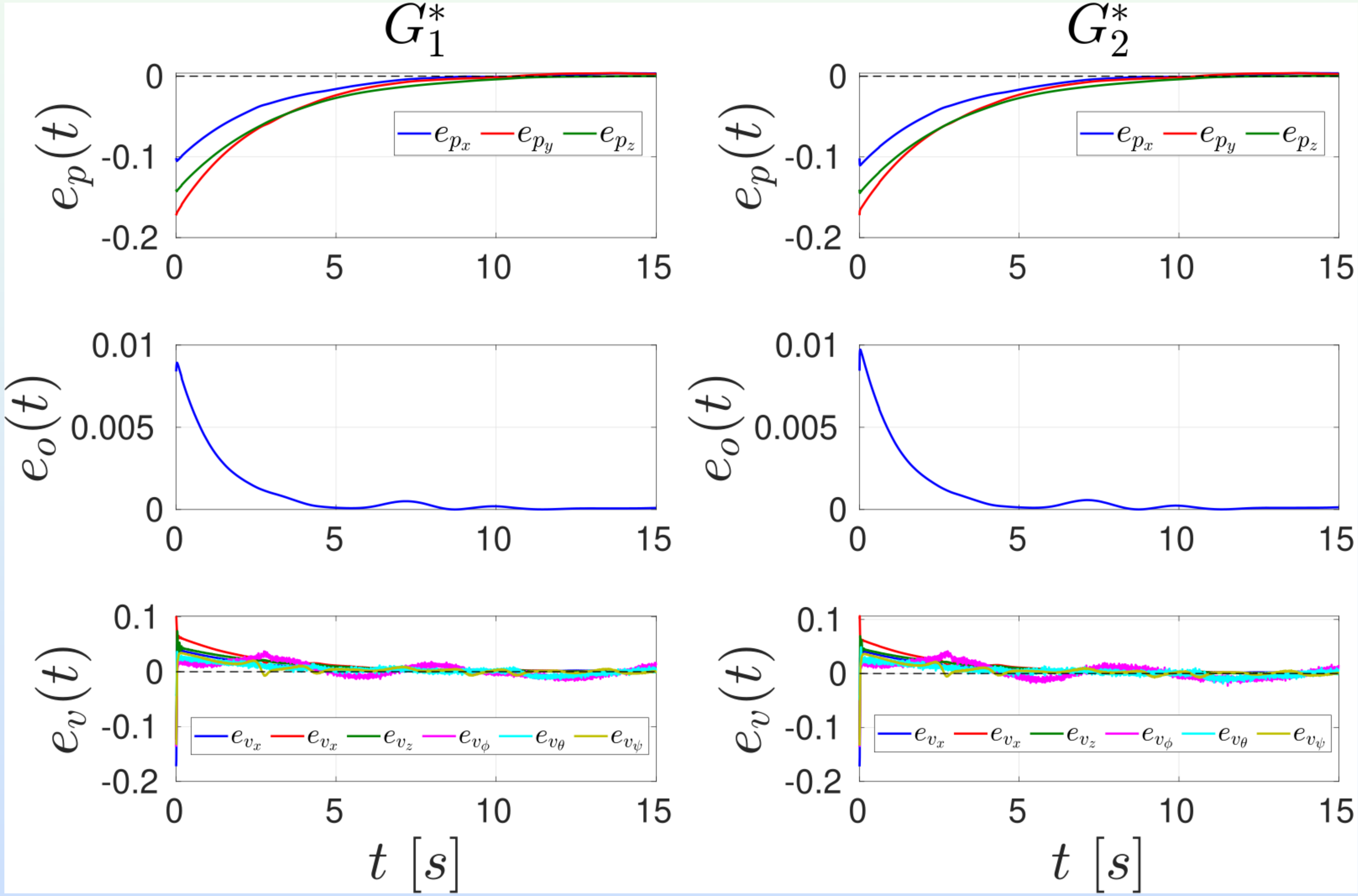
object

reference trajectory



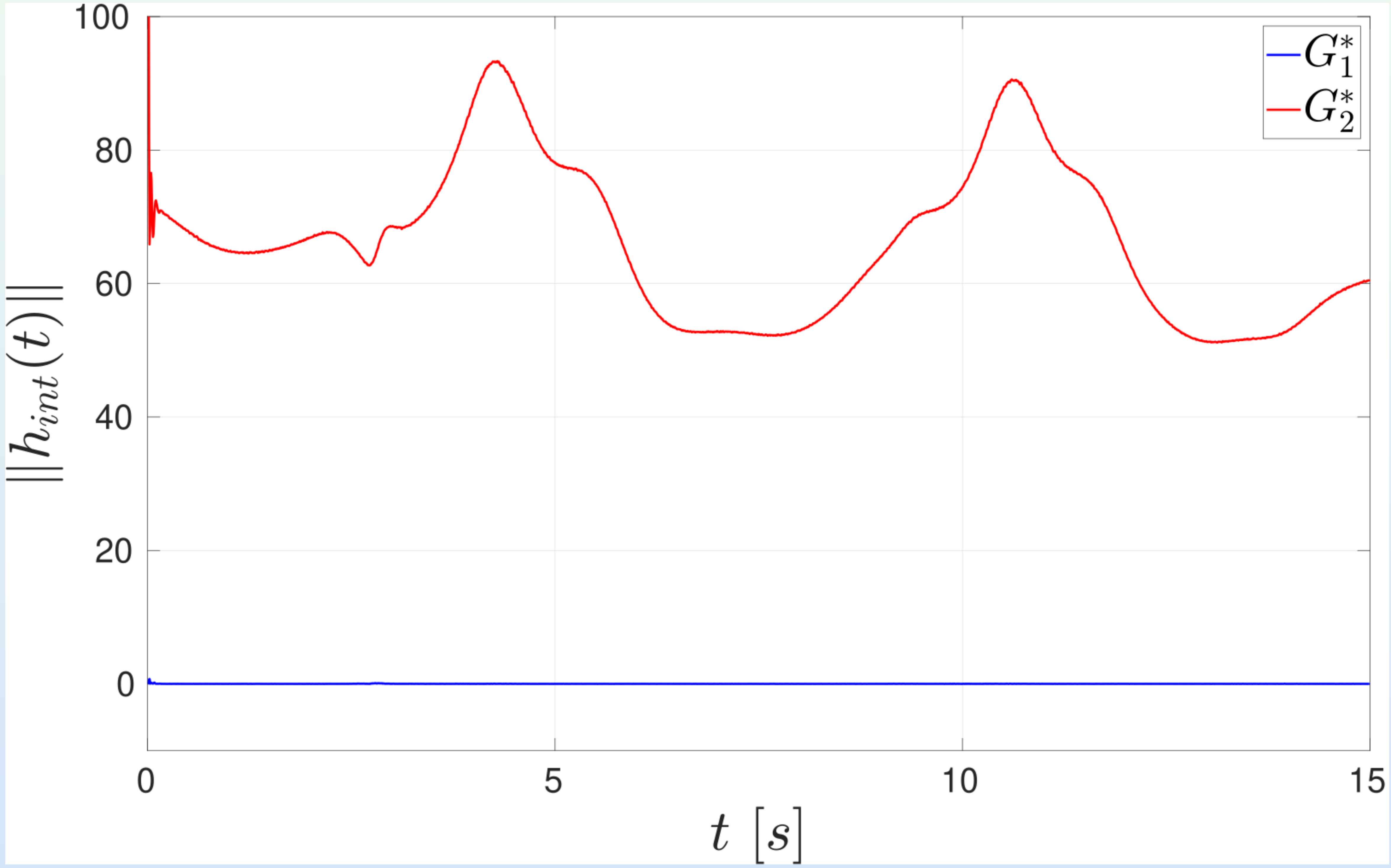
- Comparison among $\begin{cases} G_1^* = MG^\top (GMG^\top)^{-1} & \text{(proposed)} \\ G_2^* = G^\top (GG^\top)^{-1} & \text{(related works)} \end{cases}$

Both schemes achieve tracking

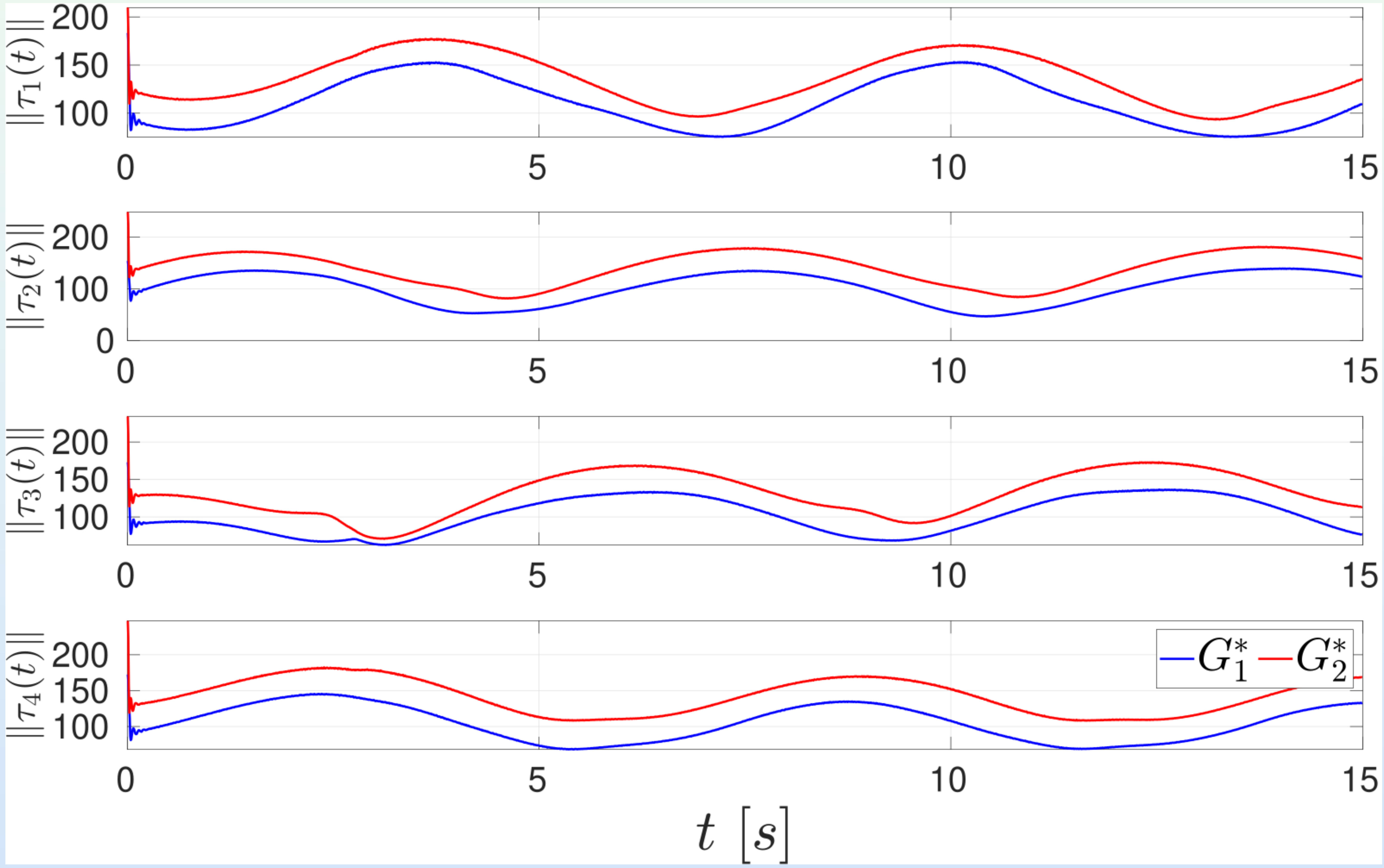


Proposed control achieves zero internal forces

Internal forces



Input norms



Conclusion and future work

Conclusion

- Cooperative manipulation via internal-force regulation
- Association of rigid cooperative manipulation with distance and bearing rigidity
- Control design that guarantees regulation of internal forces

Future work

- Extension to tethered systems
- Decentralisation and dynamic uncertainty