On the Links between Bearing-Rigid Formations and Parallel Robots

Application to Singularity Analysis of Rigid Bearing-Based Formations of Quadrotors

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ECC 2022 Workshop
Rigidity Theory applied to Dynamic Systems:
from Parallel Robots to Multi-Agent Formations

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LS2N Laboratoire des Sciences du Numérique de Nantes
CNRS
Centrale Nantes

The importance of rigidity

Rigidity of frameworks

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Framework singularities

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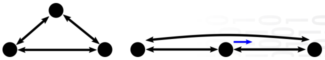

Figure: A singularity of a 3-UAV formation

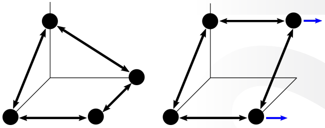

Figure: A singularity of a 4-UAV formation [Pasquetti et al. CORR 2019]

Singularities of formations

Singularities appearing in the bearing rigidity matrix

- **a huge challenge**
- Issues with singularities: **loss of controllability, of accuracy, impossibility to use pose estimation algorithms nearby**

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Determining the singularity cases stays an open problem

Introduction

The "Hidden Robot Concept": a tool for singularity analysis of rigidity matrices

- A tool made first for **visual servoing problems** [Briot et al IEEE TRO 2015, Briot et al IEEE TRO 2017]
- Then transferred to the analysis of **singularities of (bearing) rigidity matrices** [Briot & Robuffo Giordano ASME JMR 2019]

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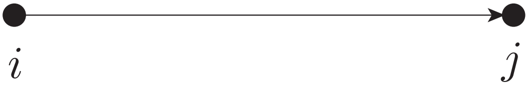
Basic idea

- The **Inverse Jacobian matrix** of a virtual parallel robot $\Rightarrow$ a basis of the Formation Rigidity Matrix
- Many tools for **finding geometric configurations** leading to singularities of line systems: Screw Theory [Hunt book 1979], Grassmann geometry [Merlet IJRR 1989], Grassmann-Cayley algebra [Kanaan et al IEEE TRO 2009]

Unidirectional bearing measurement in $SE(2)$



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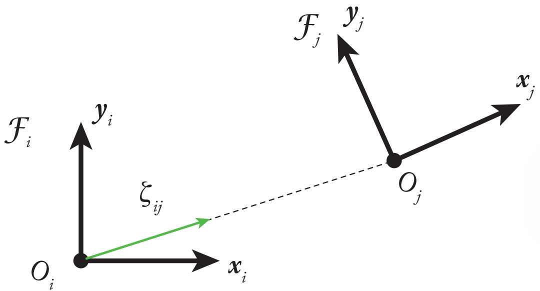

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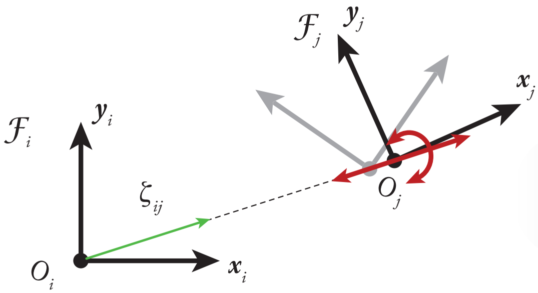

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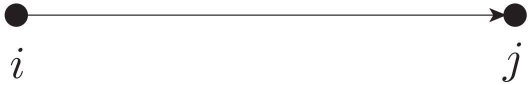

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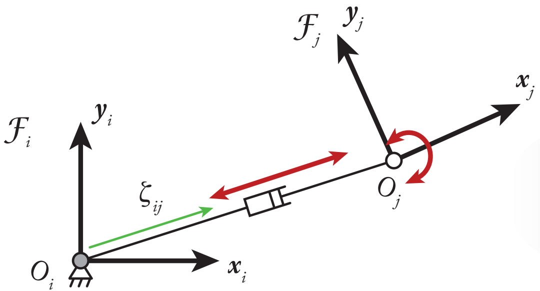

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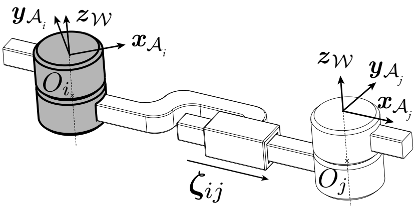

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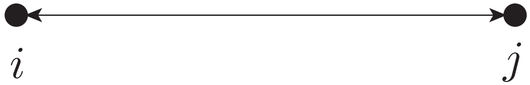

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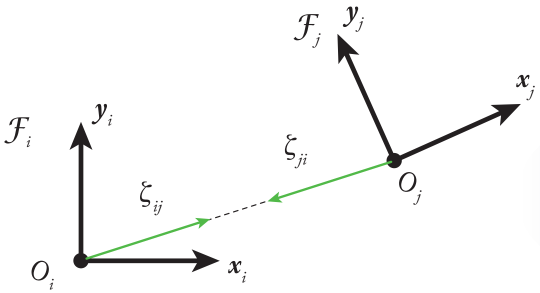

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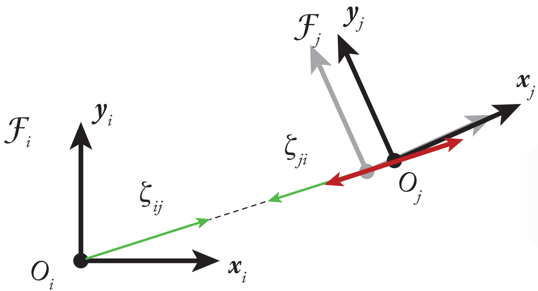

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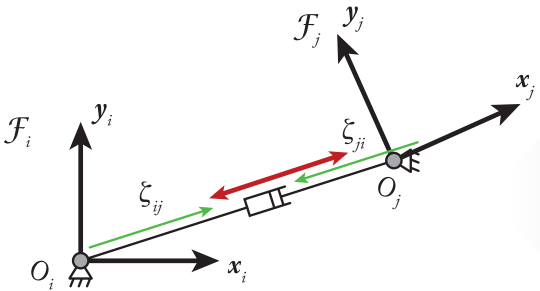

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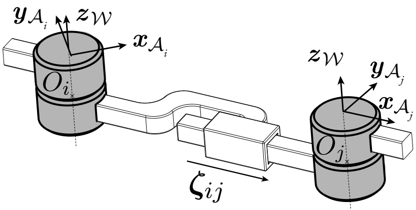
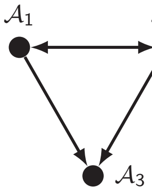


A simple graph example



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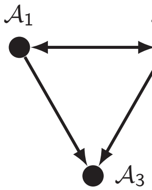

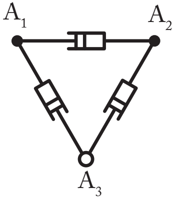

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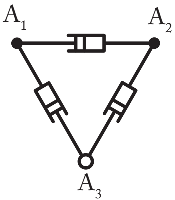

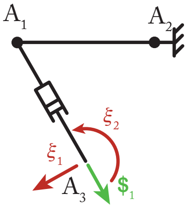

We virtually open the mechanism in A_3 + we fix one prismatic joint to avoid the scaling translation

Chain $A_2A_1A_3$

- A single twist $\$_1$ allowed in A_3:

$$\$_1 = [\mathbf{p}_{13}^T\, 0]^T \text{ translation along } \overrightarrow{A_1A_3}$$

- As a result, two wrenches constrain ξ_1 and ξ_2 the motion at A_3 ($\xi_i^T \$_1 = 0$)

$$\xi_1 = [\mathbf{p}_{13}^{\perp\,T}\, 0]^T \text{ force along } \overrightarrow{A_1A_3}^{\perp}$$

$$\xi_2 = [0\, 0\, 1]^T \text{ moment around } \boldsymbol{z}$$

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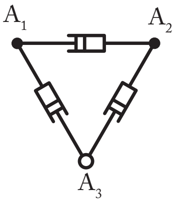

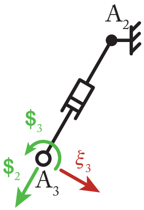

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Chain $A_2 A_3$

- Two twists $\$_2$ and $\$_3$ allowed in A_3:

$$\$_2 = [\mathbf{p}_{23}^T \, 0]^T \text{ translation along } \overrightarrow{A_2 A_3}$$

$$\$_3 = [0\,0\,1]^T \text{ rotation around } \boldsymbol{z}$$

- As a result, a single wrench ξ_3 constrains the motion at A_3 ($\xi_3^T \$_j) = 0$)

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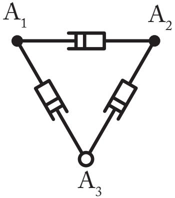

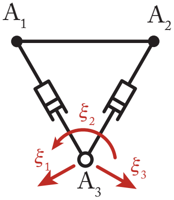

At the end, mechanism constraint wrench system $\mathcal{W}$ ($\equiv$ **basis of the rigidity matrix**)

$$\mathcal{W} = [\xi_1\ \xi_2\ \xi_3] = \begin{bmatrix} \mathbf{p}_{13}^{\perp} & \mathbf{p}_{23}^{\perp} & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix}$$

The system is rigid iff $\mathcal{W}$ is full rank

$\text{rank}(\mathcal{W}) = 3$

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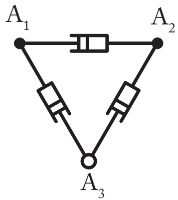

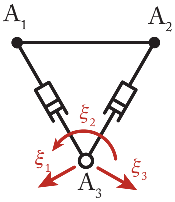

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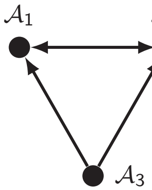
$\mathcal{W}$ and **and the rigidity matrix** are singular when locally, $\text{rank}(\mathcal{W}) < 3$, i.e. when

- $\mathbf{p}_{13}$ is colinear with $\mathbf{p}_{23}$,
- **In other words, when A_1, A_2, A_3 are aligned**.

A second graph example



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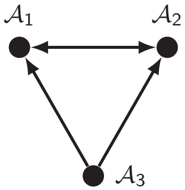

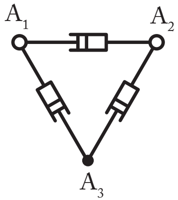

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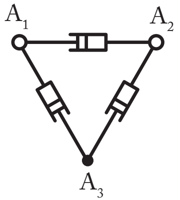

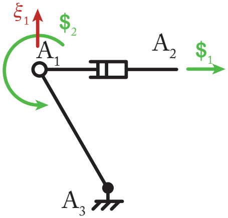

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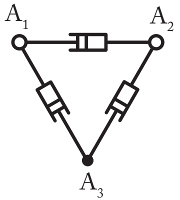

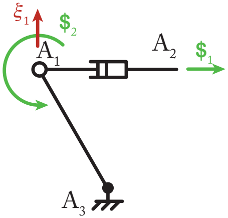

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- or also at A_2

$$\xi_1 = [\mathbf{p}_{12}^{\perp\, T} \;\; (\mathbf{p}_{12}^T \mathbf{p}_{12})]^T$$

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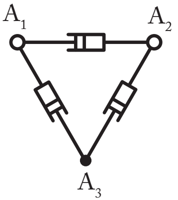

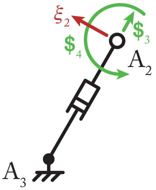

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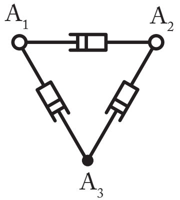

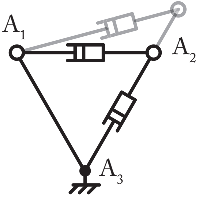

At the end, mechanism constraint wrench system $\mathcal{W}$ ($\equiv$ **basis of the rigidity matrix**)

$$\mathcal{W} = [\xi_1 \, \xi_2]$$

$\mathcal{W}$ is of rank 2:

One unconstrained motion (in the nullspace of – or reciprocal to – $\mathcal{W}$)

Hidden Robot and Singularity Analysis

Slides Julian Erskine

Strategy for a generalized singularity analysis

Objectives

- ✗ Analyse all singularities
- Analyse many singularities
- Apply to many formations
- Graph space results

What does this do ?

- Can we find all $\mathcal{S}^{\mathcal{L}}$?
- Can we find all $\mathcal{S}^{\mathcal{F}}$?
- Is $\mathcal{S}^{\mathcal{L}} \cup \mathcal{S}^{\mathcal{F}}$ comprehensive ?

Local analysis

- Apply to each agent $\mathcal{A}_i$
- Assume all agents are fixed

Figure 5.9 – Local neighbourhood of $\mathcal{A}_i$

$\mathcal{S}^{\mathcal{L}}_i$: Set of embeddings where $\mathcal{A}_i$ moves *wrt* others

Subformation analysis

- Bi-partition the formation
- Assume one partition fixed

Figure 5.10 – Subformations $\mathcal{F}_1$ and $\mathcal{F}_2$

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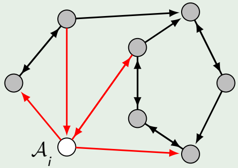

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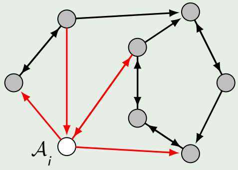

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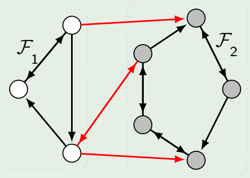

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$\mathcal{S}^{\mathcal{F}}$: Set of embeddings where $\mathcal{F}_1$ moves *wrt* $\mathcal{F}_2$

Graph edge primitives

Apply mechanical constraints to graph edges

Out edge $\mathbb{O}$

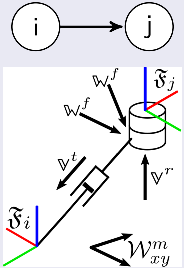

Figure 5.11 – $\mathbb{O}$ edge

2 free DOF
4 constrained DOF

In edge $\mathbb{I}$

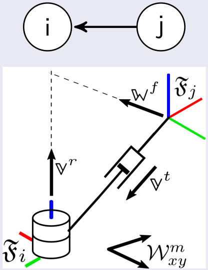

Figure 5.12 – $\mathbb{I}$ edge

2 free DOF
4 constrained DOF

Bi-direction edge $\mathbb{B}$

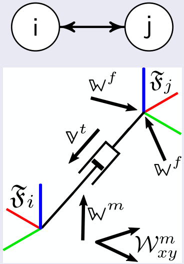

Figure 5.13 – $\mathbb{B}$ edge

1 free DOF
5 constrained DOF

Local singularities - two neighbours

Building a singularity dictionary

- Find local graph types
- Analyse the 6 hidden robots
- Use set-based analysis (e.g. $\mathcal{S}^{\mathcal{L}}_{\mathbb{BO}} \supset \mathcal{S}^{\mathcal{L}}_{\mathbb{BB}}$)

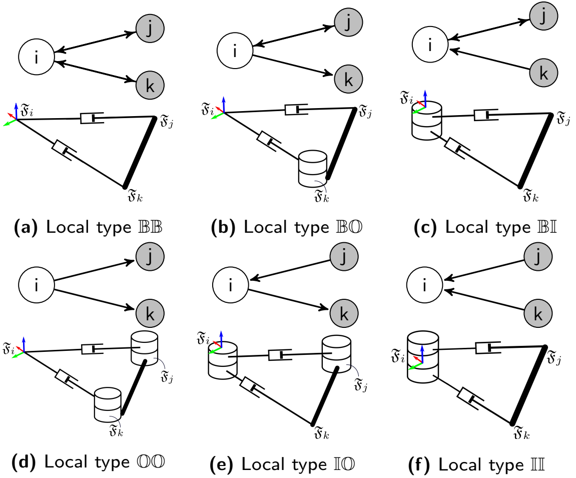

Figure 5.14 – All local formations with two neighbours

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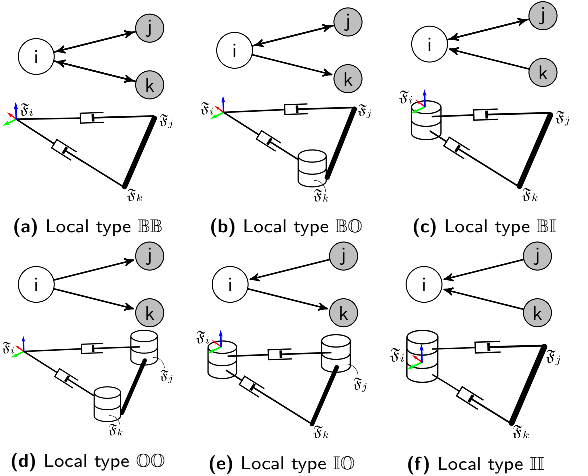
Table 3 – Singularities for of $\mathcal{A}_i$ connected to agents $\mathcal{A}_j$ and $\mathcal{A}_k$

Type	Wrenches	Singular Configuration $\mathcal{S}^{\mathcal{L}}_{\text{type}}$	Singular Twist $\mathcal{T}_{\mathcal{L}\text{type}}$
$\mathcal{L}_{\mathbb{BB}}$	$\mathcal{W}_{\mathbb{B}j} \cup \mathcal{W}_{\mathbb{B}k}$	1. $\mathcal{S}^{\mathcal{L}}_{\mathbb{BB}}$	1. $\mathcal{T}_{\mathcal{L}\mathbb{BB}}$ (see Eq. (??))
$\mathcal{L}_{\mathbb{BO}}$	$\mathcal{W}_{\mathbb{B}j} \cup \mathcal{W}_{\mathbb{O}k}$	1. $\mathcal{S}^{\mathcal{L}}_{\mathbb{BB}}$	1. $\mathcal{T}_{\mathcal{L}\mathbb{BB}}$
$\mathcal{L}_{\mathbb{BI}}$	$\mathcal{W}_{\mathbb{B}j} \cup \mathcal{W}_{\mathbb{I}k}$	1. $\mathcal{S}^{\mathcal{L}}_{\mathbb{BB}}$ 2. $\mathbf{p}_{ij}$ is vertical	1. $\mathcal{T}_{\mathcal{L}\mathbb{BB}}$ 2. $\mathbb{v}^{r}(\mathbf{z}_0, \mathbf{p}_j)$
$\mathcal{L}_{\mathbb{OI}}$	$\mathcal{W}_{\mathbb{O}j} \cup \mathcal{W}_{\mathbb{I}k}$	1. $\mathcal{S}^{\mathcal{L}}_{\mathbb{BI}}$ 2. $\mathbf{p}_{ij}$ and $\mathbf{p}_{ik}$ are horizontal	1. $\mathcal{T}_{\mathcal{L}\mathbb{BI}}$ 2. $\mathbb{v}^{r}(\mathbf{z}_0, \mathbf{c})$
$\mathcal{L}_{\mathbb{OO}}$	$\mathcal{W}_{\mathbb{O}j} \cup \mathcal{W}_{\mathbb{O}k}$	1. $\mathcal{S}^{\mathcal{L}}_{\mathbb{BO}}$ 2. $\mathbf{p}_{ij}$ and $\mathbf{p}_{ik}$ are horizontal 3. $\mathbf{p}_{jk}$ is vertical	1. $\mathcal{T}_{\mathcal{L}\mathbb{BO}}$ 2. $\mathbb{v}^{r}(\mathbf{z}_0, \mathbf{c})$ 3. $\mathbb{v}^{r}(\mathbf{z}_0, \mathbf{p}_j)$
$\mathcal{L}_{\mathbb{II}}$	$\mathcal{W}_{\mathbb{I}j} \cup \mathcal{W}_{\mathbb{I}k}$	1. $\mathcal{S}^{\mathcal{L}}_{\mathbb{BI}}$ 2. All configurations	1. $\mathcal{T}_{\mathcal{L}\mathbb{BI}}$ 2. $\mathbb{v}^{r}(\mathbf{z}_0, \mathbf{p}_j)$



(a) Local type $\mathbb{BB}$ (b) Local type $\mathbb{BO}$ (c) Local type $\mathbb{BI}$
(d) Local type $\mathbb{OO}$ (e) Local type $\mathbb{IO}$ (f) Local type $\mathbb{II}$

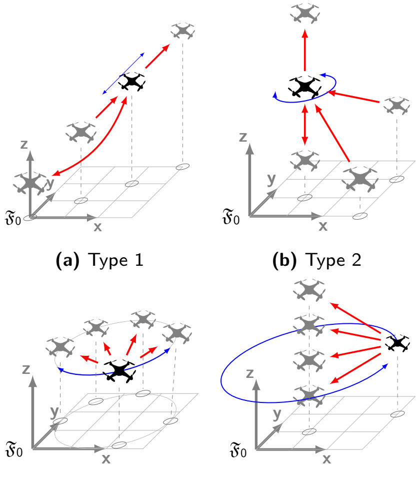
Figure 5.15 – All local formations with two neighbours

Local singularities - n neighbours

Generalization of local analysis

- Infinite number of local hidden robots
- More measurements increases constraints
- There is a closed set of local singularities

- **Singularities of all local formations of type** $\mathbb{B}^a\mathbb{O}^b\mathbb{I}^c$ **where** $a+b+c \geq 3$
  1. All edges are co-linear
  2. Only $\mathbb{I}$ edges are non-vertical
- **Singularities of local formations of type** $\mathbb{O}^b$
  3. $\mathcal{A}_i$ and $\mathcal{A}_1 \cdots \mathcal{A}_m$ lie on a horizontal circle
  4. Agents $\mathcal{A}_1 \cdots \mathcal{A}_m$ lie on a vertical line.



(a) Type 1 (b) Type 2 (c) Type 3 (d) Type 4

Figure 5.18 – All local singularities and singular motions

Subformation singularities - two edges

Local analysis is not enough

- Sufficient to imply flexibility
- Insufficient to imply rigidity

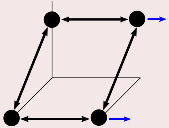


Figure 5.19 – No local singularities ($4 \times \mathcal{S}^{\mathcal{L}}_{\mathbb{BB}}$) but is clearly singular

Subformation singularities

- Assume both subformations are rigid
  - Fix one subformation
  - Other is free in translation, yaw, **scale**
- Difficult compared to local analysis

Subformation singularities - two edges

Local analysis is not enough

- Sufficient to imply flexibility
- Insufficient to imply rigidity

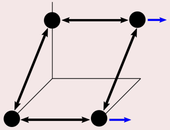


Figure 5.19 – No local singularities ($4 \times \mathcal{S}^{\mathcal{L}}_{\mathbb{BB}}$) but is clearly singular

Subformation singularities

- Assume both subformations are rigid
  - Fix one subformation
  - Other is free in translation, yaw, **scale**
- Difficult compared to local analysis

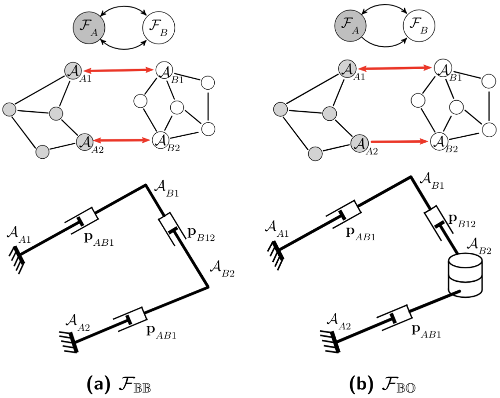


Figure 5.20 – Two subformations $\mathcal{F}_A$ and $\mathcal{F}_B$ connected by two distinct graph edges

Hidden Robot and Singularity Analysis

Table: Classification of all bi-partitioned subformation singularities with two edges

Type	Singular Configuration $\mathcal{S}^F_{type}$
$\mathbb{BB}$	1: Lines A_1B_1 and A_2B_2 intersect
	2: Lines A_1B_1 and A_2B_2 are colinear
	3: Lines A_1B_1 and A_2B_2 are vertical and superposed
$\mathbb{BO}$	1: $\mathcal{S}^F_{\mathbb{BB}}$
	2: Line A_1B_1 is vertical and intersects B_2

Subformation singularities - three edges

Subformation we can solve

- Contains $\mathcal{F}_{\mathbb{BB}}$ or $\mathcal{F}_{\mathbb{BO}}$ as subset
- Can find all singularities

Subformation we cannot solve

- $\mathcal{F}_{\mathbb{OOO}}$ and $\mathcal{F}_{\mathbb{IOO}}$
- Can find some singularities
- Cannot show that there are not others

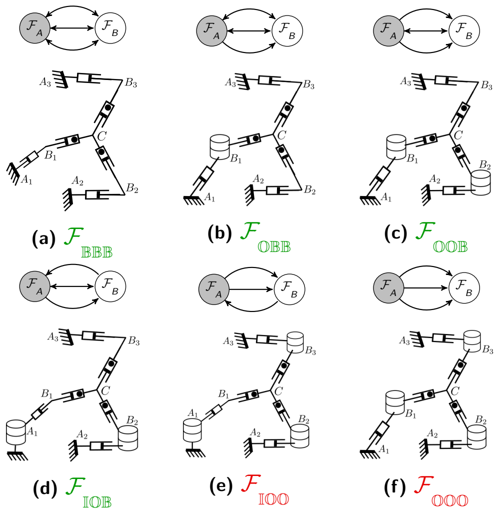


Figure 5.21 – All three-edge subformations. Prismatic joints containing a circle have coupled twist magnitudes.

Hidden Robot and Singularity Analysis

Table: Classification of all bi-partitioned subformation singularities with two edges

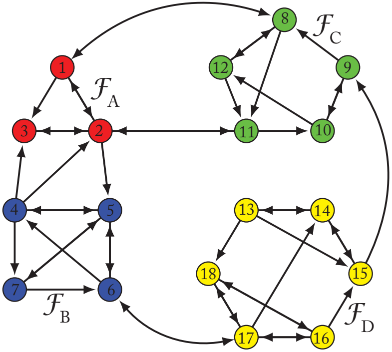
Type	Singular Configuration $\mathcal{S}^F_{type}$
$\mathbb{BBB}$	1: All lines A_iB_i intersect 2: All lines A_iB_i are colinear 3: All lines A_iB_i vertical, superposed
$\mathbb{OBB}$	1: $\mathcal{S}^F_{\mathbb{BBB}}$ 2: B_3 aligned with A_1B_1, $A_iB_i, i \in 2, 3$ superposed
$\mathbb{OOB}$	1: $\mathcal{S}^F_{\mathbb{OBB}}$ 2: Line A_3B_3 vertical, intersects B_1 or B_2
$\mathbb{IOB}$	1: $\mathcal{S}^F_{\mathbb{OBB}}$ 2: Line A_3B_3 vertical, intersects A_1 and B_2
$\mathbb{IOO}$	1: $\mathcal{S}^F_{\mathbb{IOB}}$ 2: A_1 B_2 B_3 aligned and vertical 3: All agents A_i, B_i $\forall i$ lie on a common horizontal plane
$\mathbb{OOO}$	1: $\mathcal{S}^F_{\mathbb{OOB}}$ 2: $B_i \forall i$ are vertical superposed 3: All agents A_i, B_i $\forall i$ lie on a common horizontal plane 4: F_A and F_B are bearing-congruent and lie on their common respective horizontal planes.

Hidden Robot and Singularity Analysis

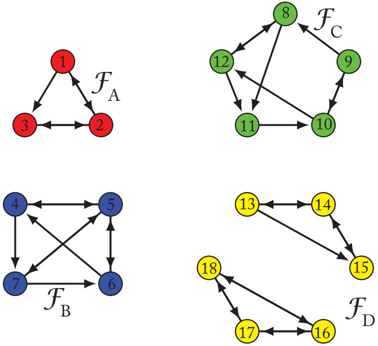
With this analysis

- Able to analyze many singularities in a formation
- Impossible to analyze all singularities for a given arbitrary formation
- BUT possibility to analyze all singularities of properly designed formations
  - simple small sub-formations in which we can easily know all singularities
  - associated together with 2 or 3 edges in order to form big-sized graphs

Case studies with 18 drones



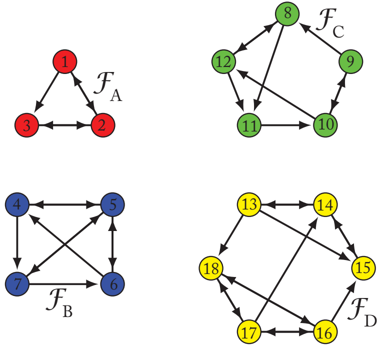
Case studies with 18 drones



Component	# of $\mathcal{S}^L$	# of $\mathcal{S}^F$
$\mathcal{S}_A$	3	
$\mathcal{S}_B$	4	
$\mathcal{S}^F_{A/B}$	-	
$\mathcal{S}_C$	5	
$\mathcal{S}^F_{AB/C}$	-	
$\mathcal{S}_D$	6	
$\mathcal{S}^F_{ABC/D}$	-	
Total	**18**	

- $\mathcal{F}_A$: $\mathbb{BB}$, $\mathbb{BO}$, $\mathbb{BI}$
- $\mathcal{F}_B$: $\mathbb{BOI} \times 3$, $\mathbb{BBB}$
- $\mathcal{F}_C$: $\mathbb{BOI} \times 3$, $\mathbb{OII}$, $\mathbb{BO}$
- $\mathcal{F}_D^*$: $\mathbb{BB} \times 4$, $\mathbb{BO}$, $\mathbb{BI}$

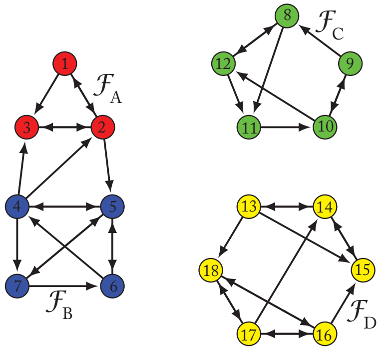
Case studies with 18 drones



Component	# of $\mathcal{S}^L$	# of $\mathcal{S}^F$
$\mathcal{S}_A$	3	0
$\mathcal{S}_B$	4	0
$\mathcal{S}^F_{A/B}$	-	
$\mathcal{S}_C$	5	0
$\mathcal{S}^F_{AB/C}$	-	
$\mathcal{S}_D$	6	1
$\mathcal{S}^F_{ABC/D}$	-	
Total	**18**	

- $\mathcal{F}_D$: $\mathbb{OII}$

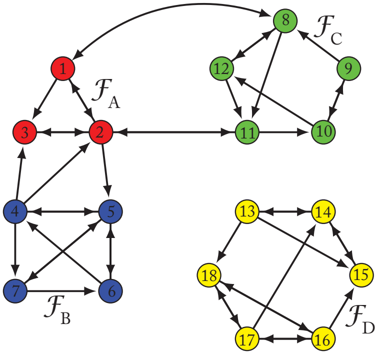
Case studies with 18 drones



Component	# of $\mathcal{S}^L$	# of $\mathcal{S}^F$
$\mathcal{S}_A$	3	0
$\mathcal{S}_B$	4	0
$\mathcal{S}^F_{A/B}$	-	1
$\mathcal{S}_C$	5	0
$\mathcal{S}^F_{AB/C}$	-	
$\mathcal{S}_D$	6	1
$\mathcal{S}^F_{ABC/D}$	-	
Total	**18**	

- $\mathcal{F}_{AB}$: $\mathbb{OOI}$

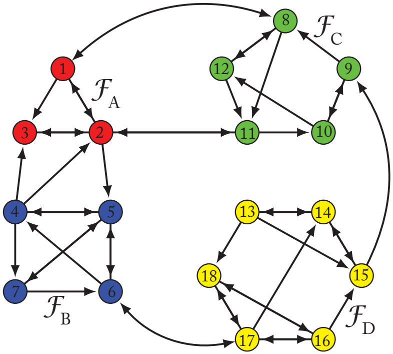
Case studies with 18 drones



Component	# of $\mathcal{S}^L$	# of $\mathcal{S}^F$
$\mathcal{S}_A$	3	0
$\mathcal{S}_B$	4	0
$\mathcal{S}^F_{A/B}$	-	1
$\mathcal{S}_C$	5	0
$\mathcal{S}^F_{AB/C}$	-	1
$\mathcal{S}_D$	6	1
$\mathcal{S}^F_{ABC/D}$	-	
Total	**18**	

- $\mathcal{F}_{ABC}$: $\mathbb{BB}$

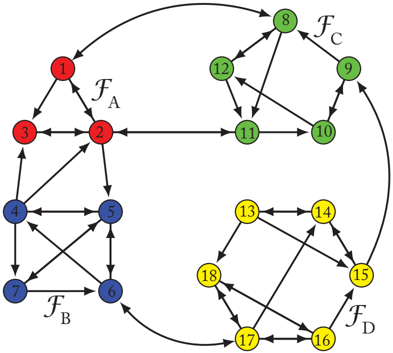
Case studies with 18 drones



Component	# of $\mathcal{S}^L$	# of $\mathcal{S}^F$
$\mathcal{S}_A$	3	0
$\mathcal{S}_B$	4	0
$\mathcal{S}^F_{A/B}$	-	1
$\mathcal{S}_C$	5	0
$\mathcal{S}^F_{AB/C}$	-	1
$\mathcal{S}_D$	6	1
$\mathcal{S}^F_{ABC/D}$	-	1
Total	**18**	**4**

- $\mathcal{F}_{ABCD}$: $\mathbb{BI}$

Case studies with 18 drones



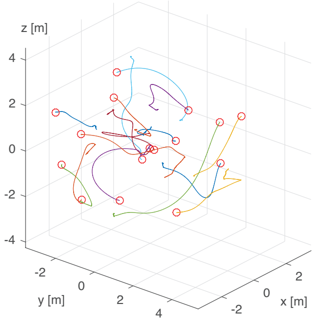
65 singularity conditions in total (*some of them redundant*):

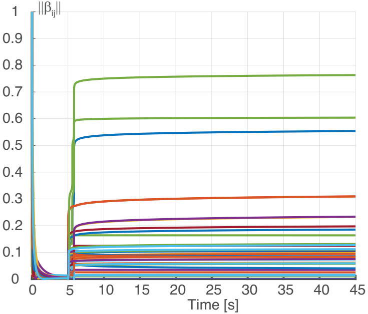
- (1, 2, 3) or (8, 9, 10) or (13, 14, 15) or (13, 14, 15) or (4, 5, 6, 7) or (8, 9, 11, 12) or (9, 10, 11, 12) or (8, 10, 11, 12) ***aligned***,
- $\mathbf{p}_{1,3}$ or $\mathbf{p}_{9,10}$ or $\mathbf{p}_{14,15}$ or ($\mathbf{p}_{4,5}$ & $\mathbf{p}_{4,7}$) or ($\mathbf{p}_{4,6}$ & $\mathbf{p}_{5,6}$) or ($\mathbf{p}_{5,7}$ & $\mathbf{p}_{6,7}$) or ($\mathbf{p}_{8,11}$ & $\mathbf{p}_{8,12}$) or ($\mathbf{p}_{9,10}$ & $\mathbf{p}_{10,12}$) or ($\mathbf{p}_{8,11}$ & $\mathbf{p}_{8,12}$) **vertical**
- lines trough (13, 18) & (14, 17) & (15, 16) ***intersect*** (in at least a point, possibly at infinity)
- ... (*other conditions of the same type*)
- (2, 3, 4, 5) or (13, 14, 15, 16, 17, 18) in ***a common plane***

⇒ Possibility to create 65 singularity indices bounded between 0 and 1

Controller for rigidity maintenance

- Similar to [Zelazo et al RSS 2012] BUT with our singularity conditions
- {0-5} seconds: initial position to singularity position (Case 1)
- After 5 seconds: Use of controller for singularity free maintenance (Case 2)





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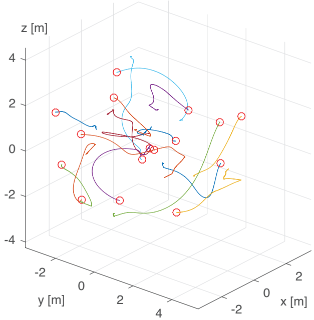

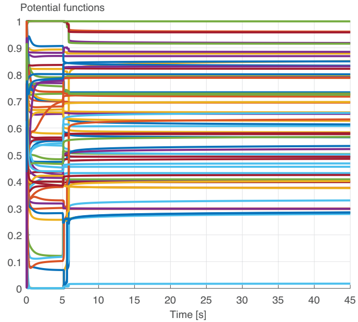

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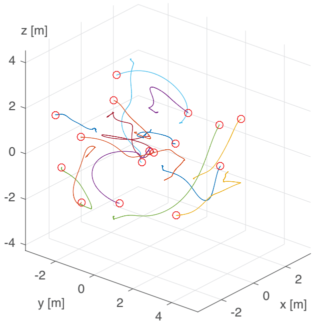

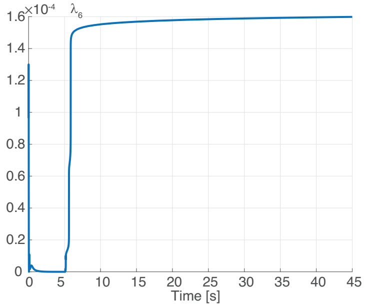

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  - Design fleets for which all singularities are known

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- Hidden robot: a tool for singularity analysis of rigidity matrices
- Problem complexity: not able to find all singularities
  - Many of them
  - Design fleets for which all singularities are known
- Controller based on this analysis
  - Behave similarly as existing singularity-avoidance controller
  - BUT we gain an intrinsic comprehension of the physics of the system